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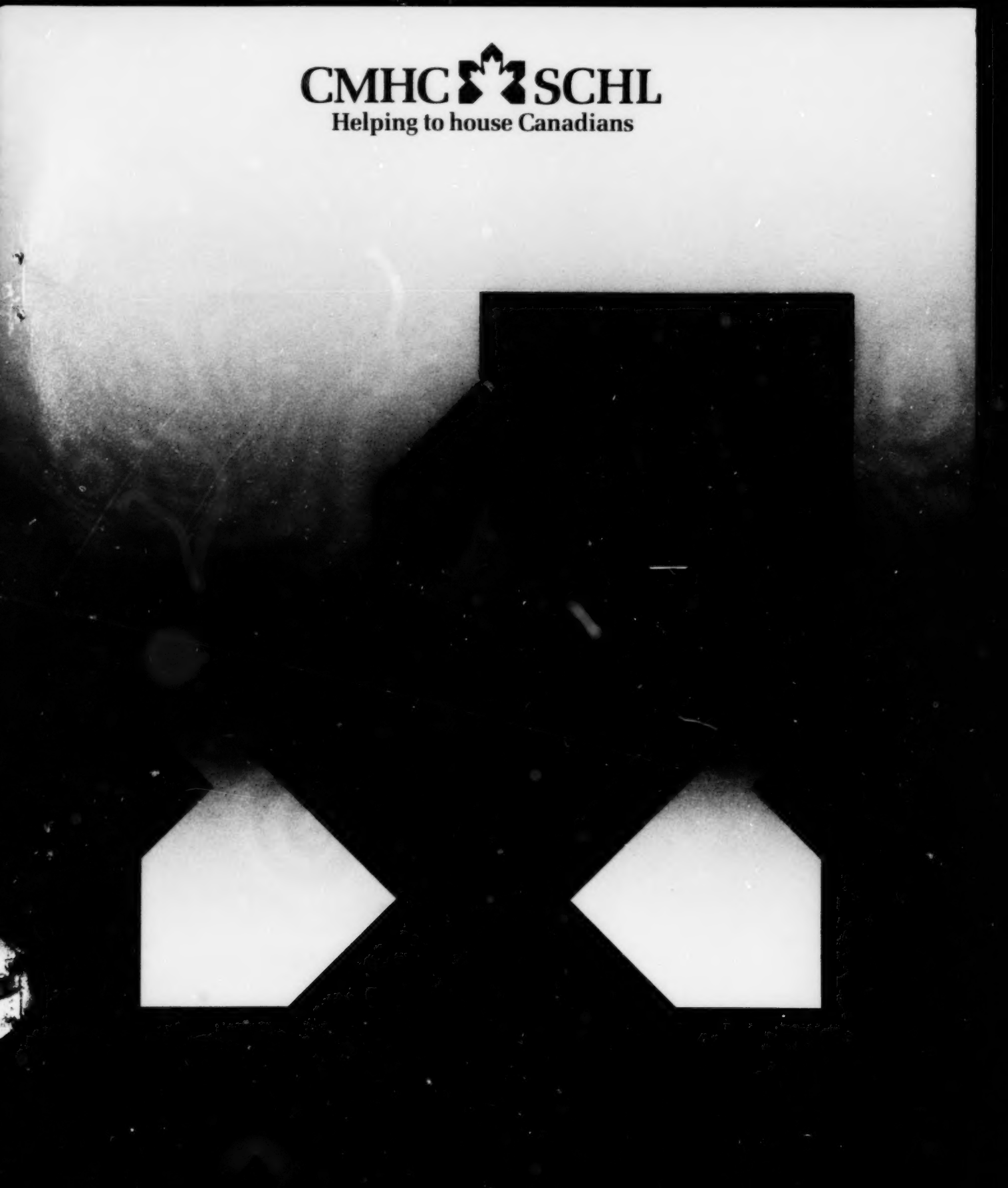

INDEX-LINKED MORTGAGES IN THE FEDERAL CO-OPERATIVE HOUSING PROGRAM:

AN ANALYSIS OF FINANCIAL RISK

92-07092/

Index-Linked Mortgages in the Federal Co-operative Housing Program

An Analysis of Financial Risk

Final Report
March 26, 1990

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I. INTRODUCTION

The Coles Group Consultants Ltd. was retained by the Canada Mortgage and Housing Corporation (CMHC) on March 7, 1990 to examine the financial risks of projects committed under the existing Federal Co-operative Housing Program, arising from rental market conditions. The performance of the current program is compared to the performance of several alternative designs.

This report presents the results of our study. A description of the assumptions and methods used for the derivation of the results is included, as well as our recommendations with respect to the existing program and the proposed alternative programs.

II. EXECUTIVE SUMMARY

This section provides a short summary of our report. The following sections provide further details regarding the assumptions and methods used to derive the results of the study.

A. Description of the Risk Model

Section III describes the operation of the computer model used for the study. The model simulates a large number of alternative economic scenarios and tabulates the frequency and magnitude of claims on the Mortgage Insurance Fund (MIF) and the Stabilization Fund (STF). The model also calculates the average cash flow and fund balance each year for the MIF and STF, as well as the present value of annual federal assistance provided to co-operatives.

The study has not examined financial problems arising from poor management and the default rates reported do not include an estimate of defaults due to mismanagement.

B. Program Alternatives

Section IV describes the current Federal Co-operative Housing Program (FCHP) and four alternative programs. The five programs under study are:

1. "Current FCHP"
   - uses an Index Linked Mortgage (ILM) with a planned 2% tilt;
   - federal assistance increases at the rate of inflation less 2%, but reduces starting in year 16 if possible;
   - initial Mortgage Insurance Fund (MIF) and Stabilization Fund (STF) premiums are each 3% of original capital cost; and
   - total STF loans are limited to 10% of the original capital cost of the project indexed at the Bank of Canada rate.
2. "Cash-feed ILM"
   - similar to the Current FCHP;
   - no initial MIF or STF premiums; and
   - unlimited annual federal assistance to cover deficits.

3. "Cash-feed EPM"
   - similar to the Cash-feed ILM;
   - financed by an equal payment mortgage (EPM).
4. "No-tilt FCHP"
   - similar to the Current FCHP;
   - the planned 2% tilt is eliminated.
5. "Market Rent Index Program"
   - similar to the No-tilt FCHP
   - federal assistance is based on the co-op's share of the mortgage payments being indexed at the rate of increase of local market rents less a 1%.

C. *Assumptions*

Section V describes the economic and operating assumptions used in the risk model. Each program was analyzed under two or three different economic scenarios. These scenarios are described in Section V. Scenario 2 is based, in part, on the parameter estimates obtained by analysis of the historical economic data. Scenarios 1 and 3 were defined by varying the economic model parameter values. The primary difference between the scenarios is the relative rates of inflation and market rent increases. Under Scenario 1, the average market rent increase exceeds the average inflation rate by 0.5% per annum. Under Scenario 2, these two factors increase at the same rate on average. Under Scenario 3 the average market rent increase is 1% per annum less than the average inflation rate.

D. *Results and Discussion*

The results of the study are described in Section VI. Results were obtained for all of the program alternatives under Scenarios 2 and 3. The Current FCHP did not generate any defaults under Scenario 1. Test runs for the alternative programs yielded similar results. For this reason, results have only been presented for the Current FCHP and Cash-feed EPM under Scenario 1.

The results indicate that the most cost effective program and the program least affected by changes in economic conditions is the Cash-feed ILM program. The Current FCHP performs relatively well in comparison to the alternative program designs that do not incorporate a cash-feed element.

The effect of removing the tilt from the ILM is to reduce total costs. However, the risk of default is substantially increased and could be considered unacceptable.

E. *Conclusions & Recommendations*

The ILM has a short history in Canada. Any conclusions about its long term viability are tentative. The key conclusions of the study are:

- the level of tilt directly affects default risk;
- the long term difference between market rent increases and inflation directly affects default risk;
- cash feed designs using ILMs are cost effective and less subject to changing economic conditions than other designs;
- maintaining a taxable STF is costly; and
- the results indicate that if market rents rise as quickly or more quickly than inflation, the current MIF and STF premium levels might be reduced (assuming that adequate provision has been made for financial problems due to poor management). However, the default risk, and the premiums necessary to ensure the solvency of the MIF and STF, increase significantly if rent increases consistently lag behind inflation, by even a small margin.

The key program design recommendations are:

- the tilt factor should be retained as protection against default;
- cash-feed programs should include a cap on federal assistance. Cash-feed programs without a ceiling on the level of federal assistance may be less cost effective than the current FCHP, due to the disincentive to minimize operating costs;
- every effort should be made to secure tax-exempt status for the Stabilization Fund; and
- further analysis should be conducted on appropriate MIF and STF premium levels if the current program design is to be retained.

III. DESCRIPTION OF THE RISK MODEL

A. *Introduction*

A specialized risk model was developed to assess the risks facing the current Federal Co-operative Housing Program and several alternative program designs. For a specified set of assumptions, the model determines the amount and frequency of claims on both the Mortgage Insurance Fund (MIF) and the Stabilization Fund (STF). The model also determines the amount of government assistance required under each of the program designs.

It was assumed that the model would only assess risks due to economic conditions and would not assess risks which could be attributed to poor management of individual co-operatives.

The risk model was implemented in the form of a Fortran computer program. The program simulates the financial operations of co-operatives under a wide range of economic conditions. Under each individual set of economic conditions (an "economic simulation") some projects will default or require ongoing assistance, while others will not experience any financial difficulties.

Each economic simulation was created by drawing random sample values from probability distributions derived for local and national inflation rates, local and national vacancy rates, local market rent increases, the prime lending rate, and the 5 year mortgage rate. A description of the probability distributions for each of the economic variables is included in Section V. A description of the methods used to develop the probability distributions is included as Appendix A.

The model measures the overall financial performance of the co-operative housing projects under each economic simulation. The results for each economic simulation were combined in tabular form to derive probability distributions for the size and number of claims on the MIF and STF. The model also generates the average cash flow and fund balances for the MIF and STF, and calculates the present value of the federal assistance.

B. *Risk Model Design Objectives*

The following were identified as necessary design objectives to ensure that the risk model provided meaningful results.

1. **The model should be able to simulate a range of future economic conditions.**

   The model should not try to predict a best estimate of future economic conditions. It should measure project financial performance over a range of different, realistic economic conditions. It was especially important to ensure that the difference between local market rents and national inflation was not constant from year to year.

2. **Important results generated by the model should be quantified in the form of probability distributions.**

   Probability distributions provide both the expected value and the variability of a given quantity. They also permit statistical testing of various characteristics of the quantity being considered.

3. **Each project should be considered on an individual basis.**

   Since the financial factors can vary considerably from project to project, it was important to measure financial performance for a wide range of projects. In fact, data was provided for most co-operatives funded through the current FCHP.

C. *Assumptions*

1. **Economic**

   One of the objectives identified in building the risk model was to consider a wide range of future economic conditions. Each important economic variable was assumed to follow a prescribed probability distribution. These distributions were estimated by analysis of historical data. The economic variables modeled were:

   - local and national inflation rates,
   - local vacancy rates,
   - local market rent increases,
   - the prime lending rate, and
   - the 5 year mortgage rate.

2. **Operating**

Assumptions were also made about the operation of the projects, including:

- calculation of occupancy charges,
- levels of rent supplement units,
- operation and tax status of the STF,
- calculation of federal assistance, and
- calculation of MIF and STF losses.

Both economic and operating assumptions are described in greater detail in the following section of the report.

D. *Operation of the Model*

Projects were treated on an individual basis for each economic simulation. All projects were subjected to the same economic conditions within each simulation. The simulation ends for a specific project when either the 50th year is reached or the project eliminates all outstanding mortgage and STF loan balances. A simulation is complete after all projects have been processed. Each group of results was derived from a separate execution of the risk model. Each execution of the model conducted 200 economic simulations.

One execution of the model proceeds through the following steps.

1. Read in the parameters specific to this execution of the model from a computer file.
2. Initialize the variables which will contain the results generated by the model.
3. Process all projects through 200 different economic simulations.

   A single economic simulation consists of the following steps.

   a. Create a 50 year projection for each of the economic variables using the appropriate distribution. Section V describes the method used to create each projection.

   b. Process each project.

      1) Read in the project data and initialize all project specific variables.

      2) Although Federal assistance ends when the mortgage is paid off in 30 to 35 year, process the project through annual financial assessments for up to 50 years to allow simulation of stabilization fund loan repayments. Mortgage payments

continue without interruption as long as the project can break even. If the project can consistently break even, it will not default and the mortgage will eventually be paid off.

If the annual cashflow is positive when an annual financial assessment is conducted, the program takes the following steps.

a) If, after year 16, the occupancy charge is less than 85% of local market rent, the occupancy charge is increased to 85% of local market rent with a corresponding reduction in the federal supply assistance.

b) If, in any year, there is an outstanding STF loan and the occupancy charge is between 85% and 100% of the local market rent, the occupancy charge is increased to a maximum of 100% of local market rent to make an STF loan repayment.

The project may be unable to maintain a positive cash flow under many economic conditions.

If the annual cashflow is negative when an annual financial assessment is conducted, the program takes the following sequence of steps.

a) The project will first try to improve its cash flow by refinancing its mortgage loan. The term of the original loan may be extended from 30 to 35 years. This refinancing is not repeated.

b) If the project is still unable to maintain a positive cash flow then it may borrow funds from the STF to finance its operating deficit. A loan may be advanced at any time. The cumulative total of loans together with outstanding interest due is limited to 10% of the initial mortgage loan amount. This initial limit is increased annually by the Bank of Canada interest rate in accordance with the STF trust indenture.

c) As a last resort, to increase revenues through higher occupancy levels, the project may increase the number of rent-geared-to-income units up to a maximum of 50% of total project units. This limit was applied to all projects regardless of the province in which the project is located.

d) Once these avenues of relief have been exhausted and the project is still unable to break even, it is forced to default on the mortgage loan. At this point, the total claim against the MIF is determined and the program calculates its present value at the mortgage loan advancement date.

The outstanding STF loans are not repayable once the project defaults on the mortgage loan. The program also calculates the present value of this amount at the mortgage loan advancement date.

3) Update the project specific result variables and distributions.

4) Proceed to the next project and start the annual assessment process again.

c. Update the simulation specific result variables and distributions

d. Proceed to the next economic simulation.

4. Compile the final results and write them to a computer file.

E. *Response Distributions*

During the compilation of final results, a number of response distributions were tabulated to measure the performance of the alternative program designs under the different economic scenarios. The statistics used to characterize these distributions are described below.

1. Distribution of Defaults

A default occurs when all levels of assistance have been exhausted and the project is still unable to maintain a positive cash flow. The distribution of defaults is determined by tabulation of the number of projects which default in each simulation. From this tabulation the mean and standard deviation of the rate of default are computed .

2. **Distribution of Claims Against the MIF**

A claim is made against the MIF when there is an outstanding mortgage loan balance at the time of default. Distributions of claims against the MIF were tabulated for each program alternative. The variables tabulated were:

a. The present value of the MIF claim as a percentage of original capital cost.

When a default occurs and a claim is made on the MIF, the amount of the claim is determined and discounted back to the mortgage advancement date. The formula used to calculate the claim amount is described in Section V.B.6.a. The present value of the MIF claim, divided by the original project capital cost, represents the percentage of capital cost which needs to be invested at the mortgage advancement date to pay the claim when it occurs. The following can be calculated from this tabulation:

- the probability of a claim against the MIF; and
- mean and standard deviation of MIF claims as a percentage of capital cost, given that a claim has been made.

The amount of the MIF claim is expressed as a percentage of capital cost so that all projects have equal weight in the analysis and to facilitate the process of setting initial premiums.

b. The year that the MIF claim occurred.

3. **Distribution of Loans Advanced by the STF**

A loan is made by the STF to a project when it is unable to maintain a positive cash flow. The loan is repayable as soon as possible. Repayment may be deferred if the project is unable to recover from its cash flow problems. The model tabulated the number of STF loans made to each project, and the period each loan was outstanding before the loan was either repaid or the project defaulted. These tabulations were used to calculate the mean rate and duration of STF loans.

Some projects may need to take another loan after having retired a previously outstanding loan. Thus, the mean rate of loans per project may exceed one. The loan rate per project is the total number of loans advanced to all projects for all simulations, divided by the the number of simulations and the number of projects.

4. **Distribution of Claims Against the STF**

If there is an outstanding STF loan when a default occurs, the outstanding loan balance is written off by the STF. This loss is discounted back to the mortgage advancement date and divided by the project's original capital cost. The resulting value is the percentage of capital cost which needs to be invested at the mortgage advancement date to pay the claim when it occurs. These tabulations yield the following statistics:

- mean and standard deviation of the size of claim, given that a claim has occured; and
- the probability of a claim against the STF.

5. **Cost of Federal Assistance**

The cost of the federal assistance as a percentage of capital cost was determined by accumulating the present value at the mortgage advancement date of all federal assistance payments and dividing by the project's original capital cost.

6. **Capital Requirements**

The average annual balance and average annual cash flow were tabulated for the MIF and STF to provide data to establish appropriate premium levels for these funds. The initial premiums required to maintain each fund's solvency was estimated using the results of these tabulations.

F. Data

Data was provided by Canada Mortgage and Housing Corporation for 318 projects, which were funded under the current FCHP between 1986 and 1989 inclusive. Projects which were serially committed were excluded.

IV. PROGRAM ALTERNATIVES

This section provides a description of the different program alternatives considered in this study. The program alternatives were defined by the Program Evaluation Division of CMHC.

A. *Current Federal Co-operative Housing Program*

The first program considered was the current Federal Co-operative Housing Program (FCHP). The details of this program are well documented elsewhere. The following paragraphs provide a brief description of the program's financing.

This program uses an Index Linked Mortgage (ILM) with a 2% tilt. Payments increase annually by the increase in the Consumer Price Index (CPI) minus 2%. The mortgage holder earns interest each year based on the mortgage's Fixed Real Rate (referred to here as the real interest rate) defined in the mortgage instrument plus the mortgage's Variable Rate which is set annually equal to the full increase in the CPI. The initial amount of the mortgage loan is up to 100% of the project's capital cost plus a 3% Mortgage Insurance Fund premium and a 3% Stabilization Fund premium.

Annual federal assistance is available to bridge the gap between total costs and the revenue derived from charging market rent for each unit. The federal assistance increases each year at the same rate as the ILM payments (CPI increase minus 2%). Starting in year 16, the federal assistance may be reduced if occupancy charges have fallen below 85% of market rent.

In addition to federal assistance, the co-operative may obtain loans from the Stabilization Fund (STF). The maximum cumulative STF loan limit is 10% of the project's original capital cost, increased by interest at the Bank of Canada interest rate.

Up to 50% of a project's units may be "rent geared to income" (RGI) units in all but three provinces. The government provides assistance to bridge the gap between the full occupancy charge for a RGI unit and the reduced income-based occupancy charge paid by the unit's occupant.

B. *Cash-feed ILM*

The next program considered has been labelled the "Cash-feed ILM".

Under this alternative program, there are no initial MIF or STF premiums. This alternative uses the same ILM as used by the FCHP.

Unlimited federal assistance is provided to each project to bridge the gap between occupancy charge revenue and total costs. While the amount of direct federal assistance is anticipated to be higher than for the current FCHP, this approach eliminates the costs incurred by the MIF and STF when a project defaults. However, it may also reduce the incentive to keep operating costs low, adding to program design problems.

The Stabilization Fund is not part of the Cash-feed ILM program design. The program design does include funding for RGI units in the same manner as the current FCHP.

C. *Cash-feed EPM*

The "Cash-feed EPM" has a similar design to the Cash-feed ILM program but uses an equal payment mortgage (EPM) rather than an ILM. The EPM is renewed every five years at market interest rates.

Federal assistance is provided to each project on the same basis as for the Cash-feed ILM program. Relative to an ILM, an EPM has high payments at the beginning of the amortization period and low payments at the end. This difference means that federal assistance is initially much higher for a project funded with an EPM than the same project funded with an ILM. This difference is, however, offset in the project's later years when an EPM-funded project can actually repay part of the federal assistance provided in its initial years.

D. *No-tilt FCHP*

The "No-tilt FCHP" program design is similar to the current FCHP. However, the 2% tilt has been removed from the increase in the mortgage payments, replacement reserves, and federal assistance.

The effect of removing the tilt is to decrease mortgage payments (and federal assistance) in the early years. The subsequent rate of increase in payments is greater than under the FCHP causing relatively higher mortgage payments (and federal assistance) in later years.

E. *Market Rent Index ILM*

The "Market Rent Index ILM" is similar to the No-tilt FCHP except for the calculation of federal assistance. The federal assistance is the same as for the No-tilt FCHP in the first year. In subsequent years, the federal assistance is equal to the full mortgage payment reduced by the project's share of the mortgage payment. The project's share of the mortgage payment is equal to its share in the previous year increased by local market rent increases minus 1%.

V. ASSUMPTIONS

Economic assumptions include assumptions about factors or variables which an individual co-operative cannot control. Operating assumptions include factors which may be controlled by CMHC to help ensure the long-term viability of the co-op program. The economic variables considered relevant for studying the risk characteristics of ILM and EPM mortgages are:

- national and local inflation rates;
- local vacancy rates;
- local rent increases;
- operating costs;
- prime lending rates; and
- five year conventional mortgage rates.

Other important factors include occupancy charge levels and loss on default. These factors are discussed in the section on operating assumptions.

A. Economic Assumptions

The following models for economic assumptions were determined using real data obtained from the Canada Mortgage and Housing Corporation and Statistics Canada.

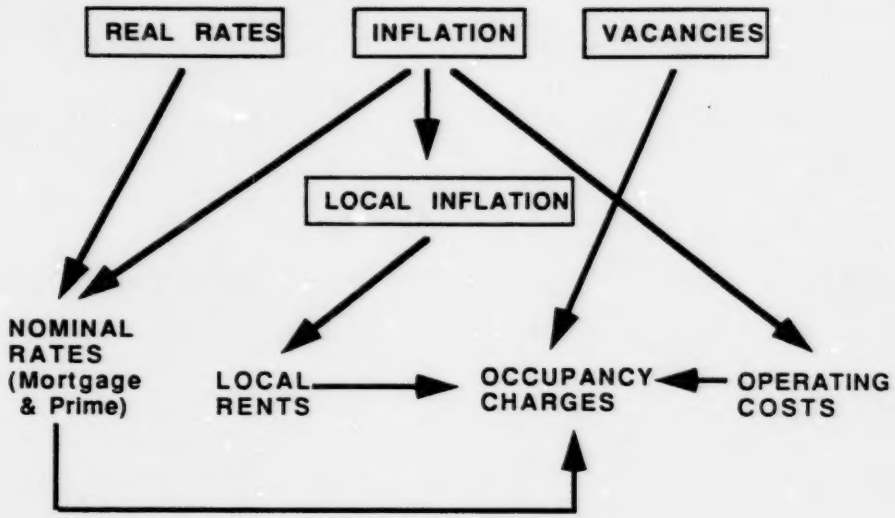
All of the raw data were in the form of proportions, with the exception of the annual inflation rate data. The annual inflation rate was determined by setting the inflation rate equal to the annual growth in the Consumer Price Index, as reported by Statistics Canada.

To ensure the accuracy of the parameter estimates and that the assumptions underlying linear regression were not violated, all variables were transformed by taking the logarithm of the variable as the independent variable under study. Thus, each economic variable has been estimated using a loglinear, or multiplicative model.

The parameter estimates for each model obtained by analysis of the historical data were used only as guidelines in setting the economic scenarios. For a more detailed description of the methods used to establish the formulas and estimate the parameters presented in this section, refer to Appendix A.

The structure of the interdependencies between the seven economic factors is illustrated in Figure V.1.

FIGURE V.1 - Interdependencies of Economic Factors



1. **Regional Data**

The variability between regions in Canada of vacancy and inflation rates made it necessary to analyze these variables on a regional basis. For the purposes of the study the regions are defined as follows:

Region	Definition
B.C.	B.C.
Alberta	Alberta
Prairies	Saskatchewan & Manitoba
North-Central Canada	northern rural regions of Ontario & Quebec
Ontario	Ontario, excluding North-Central region
Quebec	Quebec, excluding North-Central region
Atlantic	Newfoundland, Nova Scotia, P.E.I., & New Brunswick

The following cities are included in the North-Central region:

City	Province
Chicoutimi-Jonquiere	Quebec
Sudbury	Ontario
Thunder Bay	Ontario

2. Vacancy Rates

The vacancy rate data was comprised of annual vacancy rates for privately initiated apartment structures of 6 units and over, measured in October of each year for 27 cities in seven regions for the years 1969 to 1989 inclusive. The corresponding regional and national inflation rates were used to determine if a relationship exists between vacancy rates and inflation rates.

Analysis of the data showed no correlation between regional vacancy rate and regional or national inflation rates. The model for vacancy rates is:

$$V_{t_i} = \mu_i^{0.2412} \times V_{t_i-1}^{0.7066} \times V_{t_i-2}^{0.4959} \times V_{t_i-3}^{-0.4437} \times e^{\varepsilon_t}$$

where, V_{t_i} is the vacancy rate as a percentage in region i at time t, μ_i is the mean vacancy rate for region i, and ε_t is a normally distributed random variable with mean 0 and standard deviation = 0.52%.

Thus, the estimated vacancy rate for a given year in a region is defined as the weighted geometric average of the mean vacancy rate for the region and the three previous years vacancy rates for that region. The variance within each region due to error is constant between the regions. The regional means, μ_i, are:

Region	μ_i
B. C.	0.6262
Alberta	2.0967
Prairies	2.0280
North - Central	1.1267
Ontario	1.0499
Quebec	2.9755
Atlantic	2.5027

3. National Inflation Rate

The data used to determine a model for the national inflation rate is the rate of growth in the National Consumer Price Index (CPI), measured in October of each year from 1969 to 1989 inclusive. The data was provided by Statistics Canada. The model for the national inflation rate is:

$$I_t = (1 + \mu)^{1-0.7858} \times (1 + I_{t-1})^{0.7858} \times e^{\varepsilon_t} - 1$$

where, I_t is the inflation rate at time t, μ is the long term inflation rate, and ε_t is a normally distributed random variable with mean 0 and standard deviation = 1.73%. Thus the estimated inflation rate for a given year is defined as the weighted geometric average of the long term mean inflation rate, estimated at 6.9% in the study period, and the previous year's inflation rate. The historical long term inflation rate was not used as the average annual inflation rate in Scenario 2.

4. Regional Inflation Rate

Because regional rent increases are modelled as a function of regional inflation rates, it is necessary to model regional inflation rates.

The data used to determine a model for the regional inflation rate is the rate of growth in the Regional Consumer Price Index (CPI), measured in October of each year from 1969 to 1989 inclusive. The data was provided by Statistics Canada.

The national CPI is a weighted average of all the regional values for CPI. This suggests that an appropriate simulation model for national and regional inflation rates is to have separate, random series for the regional inflation rates and to define the national inflation rate as an appropriately weighted average of these separate series. The resulting model would, however, be unrealistic since the model would imply that the regional series would be independent of each other. A more realistic approach is to first determine a value for the national inflation rate for a given year, then make regional adjustments according to an appropriate model.

The data indicates that inflation rates in different regions vary together and are closely distributed around the national rate in a given year. The model for regional inflation rates is,

$$L_{ti} = (1 + I_t) \times e^{\varepsilon_{ti}} - 1$$

where L_{ti} is the inflation rate in year t for region i, I_t is the national inflation rate in year t, and ε_{ti} is a normally distributed random variable with mean 0 and standard deviation = 0.89%.

5. Market Rent Increases and Inflation

The data used to measure regional market rent increases is the growth in market rents measured in October of each year for 27 cities in 7 regions from 1984 to 1989 inclusive. The data came from two sources. For the years 1984 through 1986, the data was obtained from the Apartment Vacancy Survey provided by the Statistical Services Division. For the years 1987 through 1989, the data was obtained from the Rental Market Survey System. This data was also provided by the Statistical Services Division.

The relationship between rent increases and the inflation rate was determined using regional inflation rates rather than the national inflation rate. The model for rent increases is:

$$R_{ti} = (1 + L_{ti})^{0.8332} \times e^{\varepsilon_{ti}} - 1$$

where R_{ti} is the rent increase in year t for region i, L_{ti} is the regional inflation rate for region i in year t, and ε_{ti} is a normally distributed random variable with mean 0 and standard deviation = 2.86%. Therefore, during the study period, the local rent increase is equal to a portion of the local inflation rate. For the purposes of the risk analysis, the standard deviation of the error term was adjusted to 2.02% to remove some of the variability associated with using a small data sample. Furthermore, the exponent of 0.8332 was replaced by 1.0 for Scenario 2.

6. Prime and Mortgage Rates

The data used for the prime rate is the chartered bank prime business lending rate, measured in October of each year from 1969 to 1989 inclusive as reported by Statistics Canada. The data used for the mortgage rate is the conventional 5-year mortgage lending rate as reported by Statistics Canada.

The model for the prime rate is:

$$P_t = (1 + P_{t-1})^{0.8081} \times (1 + I_t)^{0.4274} \times (1 + I_{t-1})^{-0.1122} \times e^{\varepsilon_t} - 1$$

where P_t is the prime rate at time t, I_t is the national inflation rate at time t and e^{ε_t} is a normally distributed random variable with mean 0 and standard deviation = 1.79%. The estimated prime rate in year t is a weighted geometric average of the previous year's prime rate and the current and previous year's national inflation rate. The model for the mortgage rate is:

$$M_t = (1+M_{t-1})^{.4895} \times (1+I_t)^{0.0431} \times (1+I_{t-1})^{0.0235} \times (1+P_t)^{0.5302} \times e^{\varepsilon_t} - 1$$

where M_t is the mortgage rate at time t, I_t is the inflation rate at time t, P_t is the prime rate at time t, and e^{ε_t} is a normally distributed random variable with mean 0 and standard deviation = 0.71%. The mortgage rate in year t is a geometric average of the previous year's mortgage rate, the current and previous year's inflation rate, and the current prime rate.

7. Bank Rate of Canada

For the purposes of this study, the Bank of Canada interest rate is assumed to be equal to the prime rate minus one percentage point.

8. Operating Costs

Net operating costs for the co-operative projects in all regions are assumed to increase at the national rate of inflation plus 1%. Thus, projects are assumed to be more costly to operate as they age.

Net operating costs exclude contributions to replacement reserves and the security of tenure fund. Replacement reserve contributions increase at the national inflation rate minus 2%. Security of tenure fund contributions increase by the national inflation rate.

B. Operating Assumptions

1. Occupancy Charges

Occupancy charges may increase increase or decrease as needed to cover total operating costs net of program assistance. Occupancy charges are, however, capped at a prescribed

percentage of local market rents. The following three steps may be taken to eliminate any operating deficit before the project defaults:

a. Refinance the mortgage from 30 to 35 years,

b. Borrow funds from the Stabilization Fund, up to the remaining available loan limit, in order to reduce occupancy charges to the prescribed maximum percentage of market rents,

c. Increase the number of rent supplement units, up to 50%, if there are vacant units in the co-op.

2. Rent Supplement Units

Additional rent supplement units are available to co-ops with operating deficits, up to a maximum of 50% of all units. The minimum number of rent supplement units is 15% of total units in the co-op.

3. Treatment of the Stabilization Fund

An individual co-op may continue to draw funds from the Stabilization Fund provided the cumulative total of loans and accrued interest is kept below 10% of the initial mortgage loan amount. This initial limit is increased annually by the Bank of Canada interest rate. The solvency status of the STF is not considered when a loan is required. The loan will be granted, up to applicable limits, regardless of the balance of the fund.

Stabilization Fund loan repayments cannot cause occupancy charges to exceed 100% of local market rents. Loan repayments begin as soon as possible. Occupancy charges are kept at market rent until Stabilization Fund loans are fully repaid.

4. Tax Status of the Stabilization Fund

The Stabilization Fund is assumed to be subject to taxation. The following are assumed:

- premiums paid to the Stabilization fund are not taxed;
- all investment income is taxed at the rate of 47.18%;
- investment income includes returns from capital investments and interest paid on loans to co-ops; and
- premiums contributed by new co-op projects may not be used to advance loans to existing co-ops.

5. Decrease in Level of Federal Assistance

In year 16, the annual federal assistance is reduced by a maximum of 5% of the year 15 occupancy charge.

In subsequent years, the maximum reduction is calculated by increasing the year 16 maximum reduction by the increase in the CPI minus 2%.

The actual reduction is limited so that occupancy charges remain at or below 85% of local market rents. The level of federal assistance (before reduction) continues to be increased by the increase in the CPI minus 2%.

6. Loss on Default

The co-op continues to meet its mortgage obligations as long as occupancy charges are maintained at a break even level. If the co-op can no longer operate at a break even level, and all avenues of relief have been exhausted, then it defaults.

Upon default, the co-op is immediately purchased by the federal government for conversion to social housing. The price paid to the MIF is equal to the properties' appraised market value.

a. Loss to MIF

The loss to the MIF on default is:

MIF loss = outstanding mortgage x (1+transition expenses) – net market value

where transition expenses are 7% of the outstanding mortgage balance, and

$$\text{net market value} = \frac{\text{net cash flow}}{\text{capitalization rate}}$$

net cash flow = market rent - operating costs

operating costs = taxes + utilities + general operating costs (excludes replacement reserve + security of tenure fund)

The capitalization rate is determined using the following table:

5 Year Mortgage Rate	Capitalization Rate
8% or less	10.0%
9%	11.0%
10%	12.0%
11%	13.0%
12%	14.0%
13%	15.0%
14%	16.0%
15%	17.0%
16%	18.5%
17%	20.0%
18%	21.5%
19%	23.0%
20% or more	25.0%

b. Loss to Stabilization Fund

The loss to the Stabilization Fund is equal to the unpaid balance (principal plus accrued interest) of outstanding loans at the date of default.

8. Interest Rate for Discounting Values

The following rates are used for accruing interest and for calculating present values:

MIF/STF interest accruals	prime rate - 1% (STF is taxed)
STF loan limit accruals	prime rate - 1%
Present value calculations	prime rate - 1% (STF is taxed)
STF loan interest	prime rate
EPM mortgage interest rate	5 year conventional mortgage rate

C. Economic Scenarios

Table V.1 summarizes parameter values for the three economic scenarios used to evaluate the robustness of the ILM and EPM programs under different economic conditions. The amount by which the initial starting values differ from the mean values has a short term effect on the results. However, the effect on long term results is minimal. For this study, all economic distributions were initialized at the long term mean values shown in Table V.1.

The parameters for Scenario 2 are based mainly on the estimates obtained from direct analysis of the historical data. In this sense

Scenario 2 is a "neutral" scenario. In particular, Scenario 2 is neutral in the sense that market rent increases are assumed to keep pace with inflation. The other two scenarios use different parameter estimates than those used for Scenario 2. Scenario 1 is more "optimistic" since fewer projects are likely to default under this scenario than under either Scenario 2 or 3. Scenario 3 is considered to be relatively "pessimistic".

The estimated parameter value determined for the local market rent increases was adjusted after discussions with CMHC. The exponent of 0.8332, which represents the pull of regional inflation on the local market rents, was assigned a value of 1.0 for Scenario 2.

The following procedure was used to obtain a value for the pull of regional inflation on local market rents for Scenarios 1 and 3:

The mean of national inflation is the same as the mean of local inflation rates for all regions. Therefore, the variable I_t may be substituted for L_t in the model for local market rents after removing the error component. Thus,

$$1 + R_t = (1 + I_t)^X$$

where X is the unknown exponent. Taking the natural logarithm of both sides and solving for X gives

$$X = \frac{\ln(1 + R_t)}{\ln(1 + I_t)}$$

Substituting the desired mean national inflation rate and the mean market rent increase rate (7.0 and 6.0 respectively) and solving gives X = 0.8612 for Scenario 3. Similarly, for Scenario 1, the pull of regional inflation should be set to 1.1223 to obtain a mean local rent increase of 4.5% when the mean national inflation rate is 4.0%.

Table V.1 - Summary of Economic Scenarios

	Scenario 1	*Scenario 2*	*Scenario 3*
National Inflation			
Mean	4.00%	5.00%	7.00%
Standard Deviation	1.41%	1.73%	2.00%
Pull factor	0.3	0.2142	0.1
Regional Inflation			
Standard Deviation	0.77%	0.89%	1.00%
Vacancies			
Mean	regional mean * .8	regional mean	regional mean * 1.2
Standard Deviation	same	0.52%	same
Pull factor	-	-	-
Market Rents			
Mean	4.5%	5.0%	6.0%
Pull of Regional Inflation	1.1223	1	0.8612
Standard Deviation	1.73%	2.02%	2.24%
Co-op Vacancies	same	rent supp full	same
	regional mean * .7	private market for others	regional mean * 1.3
Stabilization Fund Trigger	110% of market	105% of market	100% of market
	No rent induced vacancies. If co-op has exhausted Stab. Fund assistance and still above trigger then fill up to 50% as rent supp. units. Default if still above trigger		
Mortgage Rate Standard Deviation	same	0.71%	same
Prime Rate Standard Deviation	same	1.79%	same

VI. RESULTS

This section of the report presents the results of the study. The charts referred to in the text are included at the end of this section.

Results are presented for the four different program designs described in Section IV using the three economic scenarios described in Section V.

We have also presented results of Phase I of our study for comparison to the results of the 1985 Wyatt study. These results are included for illustrative purposes. The Phase I results were derived using the methods and assumptions described in the 1985 Wyatt study.

A. Distribution of Defaults

Table VI.1 shows the statistical summary for the distribution of defaults for all program alternatives, except for the two cash-feed programs. Under the cash-feed programs, the amount of available federal assistance is unlimited, therefore, a default is not possible.

Table VI.1 indicates that the mean rate of default is significantly greater for the No-tilt FCHP than all other programs. This result is the same for all economic scenarios. The Current FCHP and the Market Rent Index alternatives are comparable, with the Current FCHP faring better under Scenario 2, but worse under Scenario 3.

The mean year of default does not vary significantly between the program alternatives nor between economic scenarios. The only exception being that no defaults occurred when using Scenario 1 assumptions.

The Phase I results show that the projected default rate under the Current FCHP is significantly less than the default rate of the ILM program described in the 1985 Wyatt study.

The 1990 Data results in Phase I were obtained by modifying the 1990 data to reflect the real rate of return anticipated in the 1985 study rather than the actual real rate of return defined at the project commitment date. This data was then analyzed using the 1985 assumptions and the 1985 risk model computer program updated to reflect the current FCHP.

Table VI.1 - Default Statistics

	YEAR OF CLAIM		RATE OF DEFAULT	
	Mean	Std. Dev.	Mean	Std. Dev.
Phase I - 1985 Assumptions				
• 1985 Data	25.9	5.6	5.0%	5.6%
• 1990 Data	27.6	8.4	1.2%	2.6%
Phase II - Scenario 2				
• Current Program	29.2	4.7	0.7%	1.5%
• No Tilt	27.7	5.1	19.9%	19.4%
• Market Rent Index	28.2	5.8	1.4%	2.3%
Phase II - Scenario 3				
• Current Program	24.7	6.6	47.1%	20.1%
• No Tilt	19.8	5.9	90.2%	13.5%
• Market Rent Index	26.3	6.0	46.8%	18.2%
Phase II - Scenario 1				
• Current Program	0.0	0.0	0.0%	0.0%

B. Claims Against the MIF

1. MIF Claims by Year

Figure VI.2 shows the probability distribution of MIF claims by year for the current FCHP under Scenario 2. The mean year of claim is 29.2 years.

2. MIF Claims as a Percentage of Capital Cost

Figure VI.3 shows the probability distribution of MIF claims as a percentage of original capital cost, given that a claim occurs, for the Current FCHP under Scenario 2. Table VI.2 shows the relevant statistics for this distribution for all the program alternatives, excluding the cash-feed programs. Note that in Table VI.2 the Mean Claim is not conditional on a claim being made as in Figure VI.3

Under Scenario 2, the probability of no MIF claims is approximately the same for the Current FCHP and the Market Rent Index alternative, at 99.6% and 99.2% respectively. The mean MIF claim is very small for these two programs at 0.1%

and 0.2% respectively. The probability of no MIF claims for the No-Tilt FCHP is significantly lower under Scenario 2 at 90.6%. The corresponding mean MIF claim at 1.6% is much greater than the Current FCHP and the MRI alternative.

Under Scenario 1, the probability of a MIF claim was zero for the Current FCHP.

Under Scenario 3, significant MIF claims were incurred for all program alternatives. The probability of a MIF claim was much greater as was the mean MIF claim.

Table VI.2 - MIF Claims as a Percentage of Capital Cost

	Mean Claim	Std.Dev. of Claim	Probability of no claim
Phase I - 1985 Assumptions			
• 1985 Data	0.5%	3.0%	96.0%
• 1990 Data (adjust op. costs)	0.1%	1.1%	99.2%
Phase II - Scenario 2			
• Current Program	0.1%	1.1%	99.6%
• No Tilt	1.6%	6.8%	90.6%
• Market Rent Index	0.2%	2.4%	99.2%
Phase II - Scenario 3			
• Current Program	5.8%	12.4%	66.5%
• No Tilt	21.4%	19.8%	24.0%
• Market Rent Index	7.1%	15.0%	67.4%
Phase II - Scenario 1			
• Current Program	0.0%	0.0%	100.0%

C. *STF Loans*

Table VI.3 shows the mean rate of loans per project and the average length of those loans under the three scenarios. The STF was not part of the design for either of the cash-feed programs.

The Current FCHP did not require any STF loans under Scenario 1. All of the non-Cash-feed programs, however, did use STF loans under Scenarios 2 and 3. Under Scenario 2, the Current FCHP required fewer loans than either the No-tilt FCHP or the Market Rent Index program. Under Scenario 3, however, loans were more numerous for the Current FCHP than the other two programs, while their duration was shorter.

Table VI.3 - STF Loan Summary

	Rate per Project	Average Duration
Phase II - Scenario 2		
• Current Program	0.2	4.1
• No Tilt	1.2	7.6
• Market Rent Index	0.4	10.4
Phase II - Scenario 3		
• Current Program	1.7	9.6
• No Tilt	1.3	11.8
• Market Rent Index	1.3	17.2
Phase II - Scenario 1		
• Current Program	0	0

D. Claims Against the STF

Figure VI.4 shows the probability distribution of STF claims as a percentage of capital cost, given that a claim has been made, for the Current FCHP under Scenario 2. Table VI.4 shows the relevant statistics for this distribution for all the program alternatives, except for the cash-feed programs. The values in Table VI.4 are not conditional on a claim being made.

The probability of no STF claims is similar to the probability of no MIF claims.

As shown in Figure VI.4 the minimum claim made against the STF is 10% of the initial capital cost of a project. The operating assumptions define STF loan limits to be 10% of initial capital cost indexed to the date of the loan. However, because of taxation of the STF, the present value of claims made against the STF will generally be greater than or equal to 10% of the initial capital cost. For example, assume the STF loan limit increases at the the annual rate of 10% over twenty years (i.e. Bank of Canada rate = 10%). Suppose the interest earned on the STF balance is: prime rate - 1% = 10%, and interest income of the STF fund is taxed at the rate of 47.18%. The net discount rate for the STF is 5.282% ((prime rate - 1%) x (1 - tax rate)). Therefore, the loan limit will increase more quickly than the rate used to calculate the present value of outstanding loans at default (the STF claim amount) resulting in a STF claim present value of more than 10% of capital cost.

Table VI.4 - STF Claims as a Percentage of Capital Cost

	Mean Claim	Std. Dev. of Claim	Probability of no claim
Phase I - 1985 Assumptions			
• 1985 Data	0.3%	1.2%	95.6%
• 1990 Data (adjust op. costs)	0.1%	0.5%	99.1%
Phase II - Scenario 2			
• Current Program	0.1%	2.2%	99.5%
• No Tilt	4.8%	10.9%	82.0%
• Market Rent Index	1.4%	7.8%	96.3%
Phase II - Scenario 3			
• Current Program	15.2%	22.4%	57.5%
• No Tilt	25.5%	16.2%	11.2%
• Market Rent Index	27.4%	31.5%	41.7%
Phase II - Scenario 1			
• Current Program	0.0%	0.0%	100.0%

The Current FCHP and the Market Rent Index program alternative both experienced significantly fewer claims than the No-tilt FCHP.

E. *Cost of Federal Assistance*

The present value of federal assistance was calculated as a percentage of original capital cost for each of the program designs. Table VI.5 shows the results of these calculations.

The present value of federal assistance is affected by a number of factors: default rates, program design (including the timing of assistance payments) and economic factors.

The No-Tilt program has the lowest overall Federal Assistance costs because of the lower financing costs that result from removing the tilt factor.

Federal assistance for the current FCHP and the No Tilt design is lower under Scenario 3 than under Scenario 2 due to the higher default rates under Scenario 3. However, when considering overall cost, lower federal assistance costs are more than offset by higher MIF and STF claims.

Table VI.5 - Federal Assistance as a Percentage of Capital Cost

	Supply Assistance
Phase I - 1985 Assumptions	
• 1985 Data	31.7%
• 1990 Data (adjust op. costs)	38.4%
Phase II - Scenario 2	
• Current Program	43.2%
• Cash Feed ILM	36.6%
• Cash Feed EPM	54.6%
• No Tilt	29.9%
• Market Rent Index	38.2%
Phase II - Scenario 3	
• Current Program	42.7%
• Cash Feed ILM	40.3%
• Cash Feed EPM	57.6%
• No Tilt	21.0%
• Market Rent Index	44.4%
Phase II - Scenario 1	
• Current Program	41.3%
• Cash Feed ILM	12.0%

Under both Scenarios 2 and 3, the Cash-feed ILM program has lower federal assistance than the Cash-feed EPM program. This is due to differences in the timing of federal assistance payments. Under the Cash-feed ILM program, assistance payments are spread-out over the term of the mortgage while EPM program assistance is concentrated in the early years.

The Cash-feed ILM program also required less assistance than the Current FCHP under all three scenarios. The Cash-feed ILM program was designed to provide only enough assistance to ensure an individual project reached break-even. However, the design of the Current FCHP may result in a project receiving more assistance in some years than is necessary to reach break-even.

F. *Total Costs*

Table VI.6 summarizes the total costs required for each of the program alternatives under the different economic scenarios.

The total cost is defined as the sum of the present values (as a percent of capital cost) of supply assistance, MIF claims and STF claims less the applicable premiums for the MIF and STF.

For most programs, the supply assistance is large under Scenario 2 while the risk of default is relatively small. However, the risk of default is significantly greater for the No-tilt Program. This is more than offset by lower federal assistance. Thus, the most cost effective program under Scenario 2 was the No-tilt FCHP.

The Cash-feed ILM , however, was the most cost effective program under Scenario 3. The program experiencing the greatest increase in total costs when comparing Scenarios 2 and 3 was the Market Rent Index program. The results suggest that the Cash-feed ILM program is the most robust, as well as being one of the most cost effective.

Table VI.6 - Net Program Costs as a Percentage of Capital Costs

	Supply Assistance	Initial Premiums	Net Cap. Amt.	MIF Claims	STF Claims	Total Cost
Phase I - 1985 Assumptions						
• 1985 Data	31.7%	(6.0%)	25.7%	0.5%	0.3%	26.5%
• 1990 Data (adjust op. costs)	38.4%	(6.0%)	32.4%	0.1%	0.1%	32.6%
Phase II - Scenario 2						
• Current Program	43.2%	(6.0%)	37.2%	0.1%	0.1%	37.4%
• Cash Feed ILM	36.6%	0.0%	36.6%	0.0%	0.0%	36.6%
• Cash Feed EPM	54.6%	0.0%	54.6%	0.0%	0.0%	54.6%
• No Tilt	29.9%	(6.0%)	23.9%	1.6%	4.8%	30.3%
• Market Rent Index	38.2%	(6.0%)	32.2%	0.2%	1.4%	33.8%
Phase II - Scenario 3						
• Current Program	42.7%	(6.0%)	36.7%	5.8%	15.2%	57.7%
• Cash Feed ILM	40.3%	0.0%	40.3%	0.0%	0.0%	40.3%
• Cash Feed EPM	57.6%	0.0%	57.6%	0.0%	0.0%	57.6%
• No Tilt	21.0%	(6.0%)	15.0%	21.4%	25.5%	61.9%
• Market Rent Index	44.4%	(6.0%)	38.4%	7.1%	27.4%	72.9%
Phase II - Scenario 1						
• Current Program	41.3%	(6.0%)	35.3%	0.0%	0.0%	35.3%
• Cash Feed ILM	12.0%	0.0%	12.0%	0.0%	0.0%	12.0%

However, this study does not account for any effects due to disincentive to maintain reasonable operating costs.

G. *Capital Requirements*

For the current FCHP the initial MIF and STF premiums are both 3% of capital cost. Under Scenario 2 the premiums required to maintain a positive balance for both funds are each less than 1%. If the current premiums were reduced there would be a corresponding reduction in the federal assistance required.

Figure VI.5 illustrates the average MIF balance by year under Scenario 2 when the initial premium is set at 3% and at 1%. Figure VI.6 illustrates the average MIF balance under Scenario 3 when the initial premiums are set at 3% and 4.25%. Figure VI.7 shows the cash flow of the MIF under Scenarios 2 and 3.

Similarly, Figures VI.8 to VI.10 illustrate the capital requirements and cashflow for the STF for the current program under scenarios 2 and 3.

Thus, under Scenario 3, the premiums required to sustain a positive balance in the MIF and STF are both greater than 3%. The MIF becomes insolvent in year 23 if the premium is set at 3%, while the STF becomes insolvent in year 14 with a 3% initial premium.

Figure VI.1

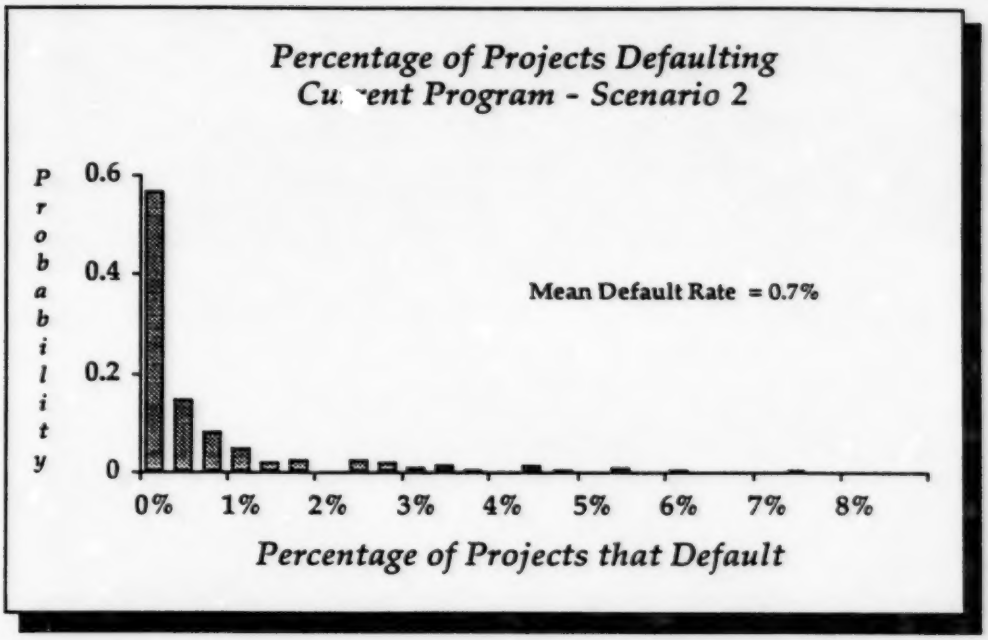


Figure VI.2

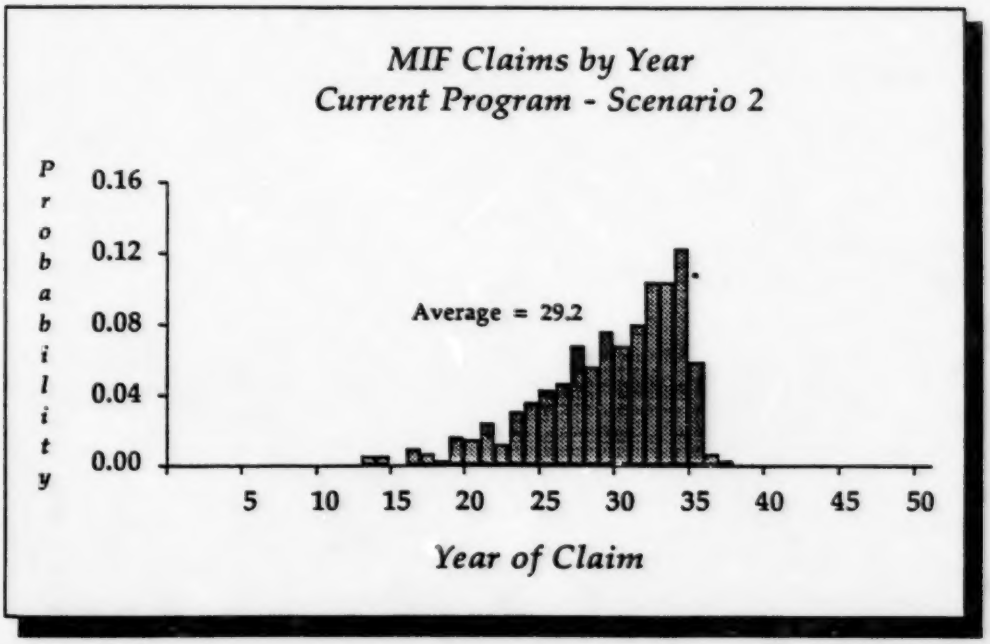


Figure VI.3

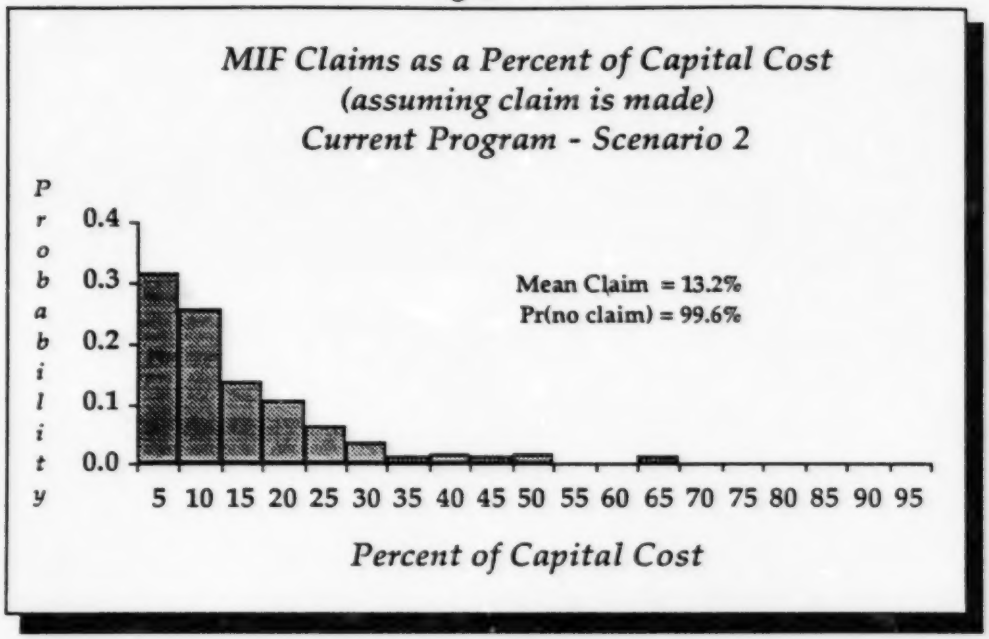

MIF Claims as a Percent of Capital Cost
(assuming claim is made)
Current Program - Scenario 2
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 13.2%
Pr(no claim) = 99.6%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

Figure VI.4

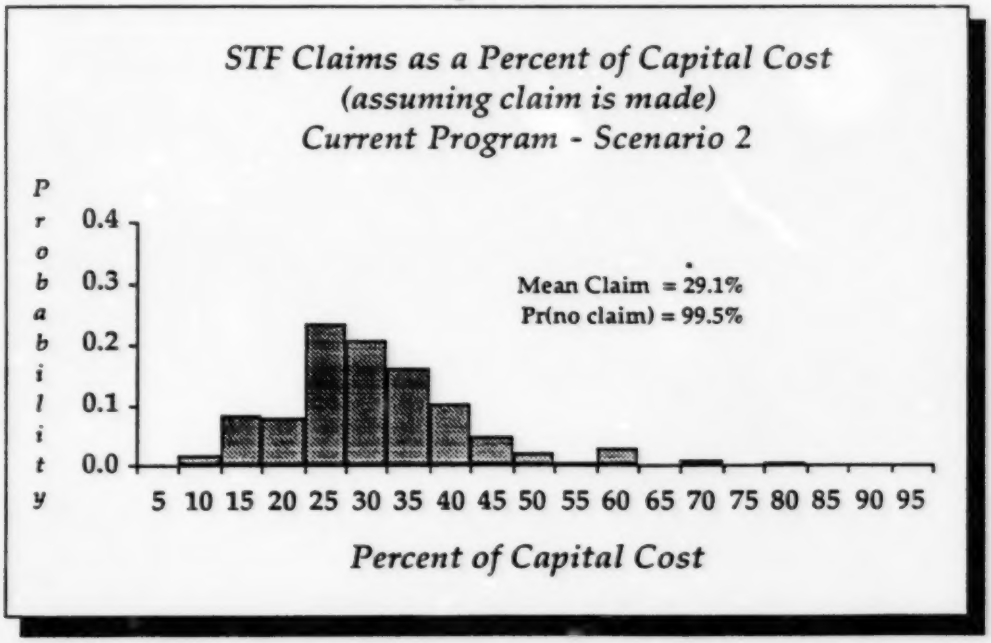

STF Claims as a Percent of Capital Cost
(assuming claim is made)
Current Program - Scenario 2
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 29.1%
Pr(no claim) = 99.5%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

Figure VI.5

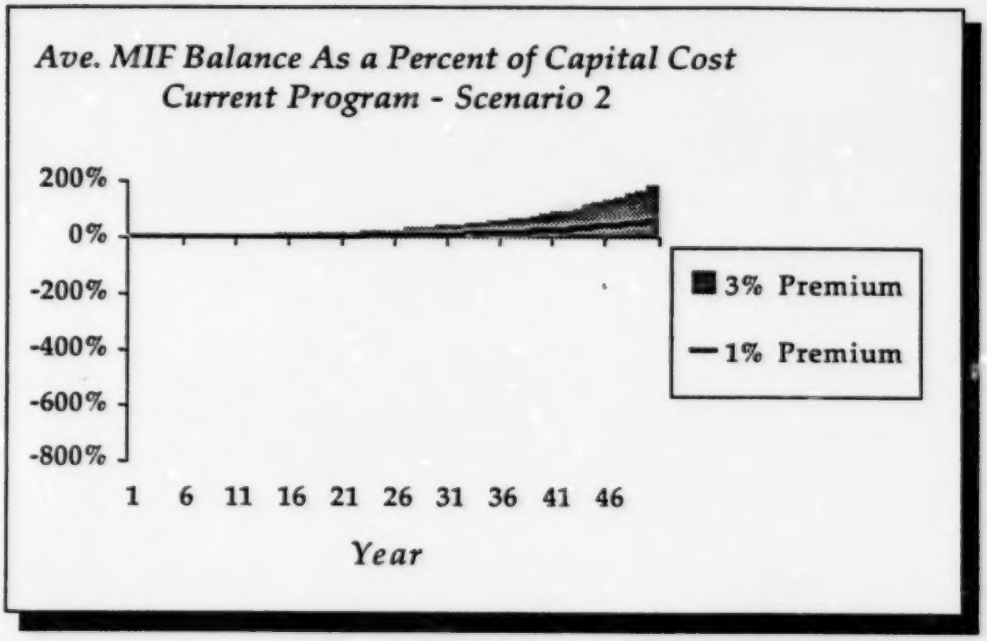


Figure VI.6

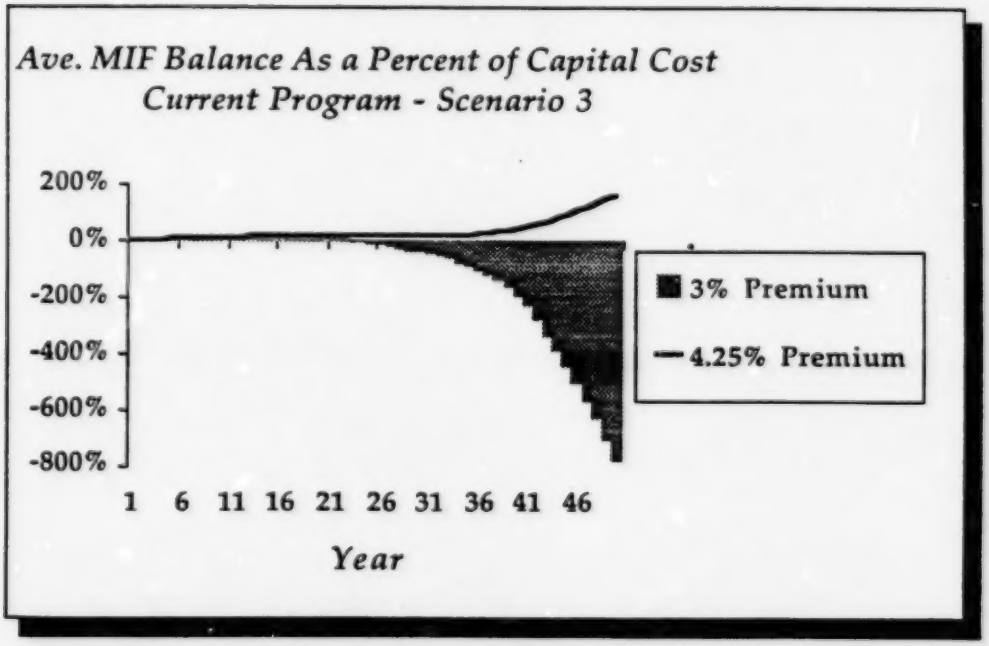


Figure VI.7

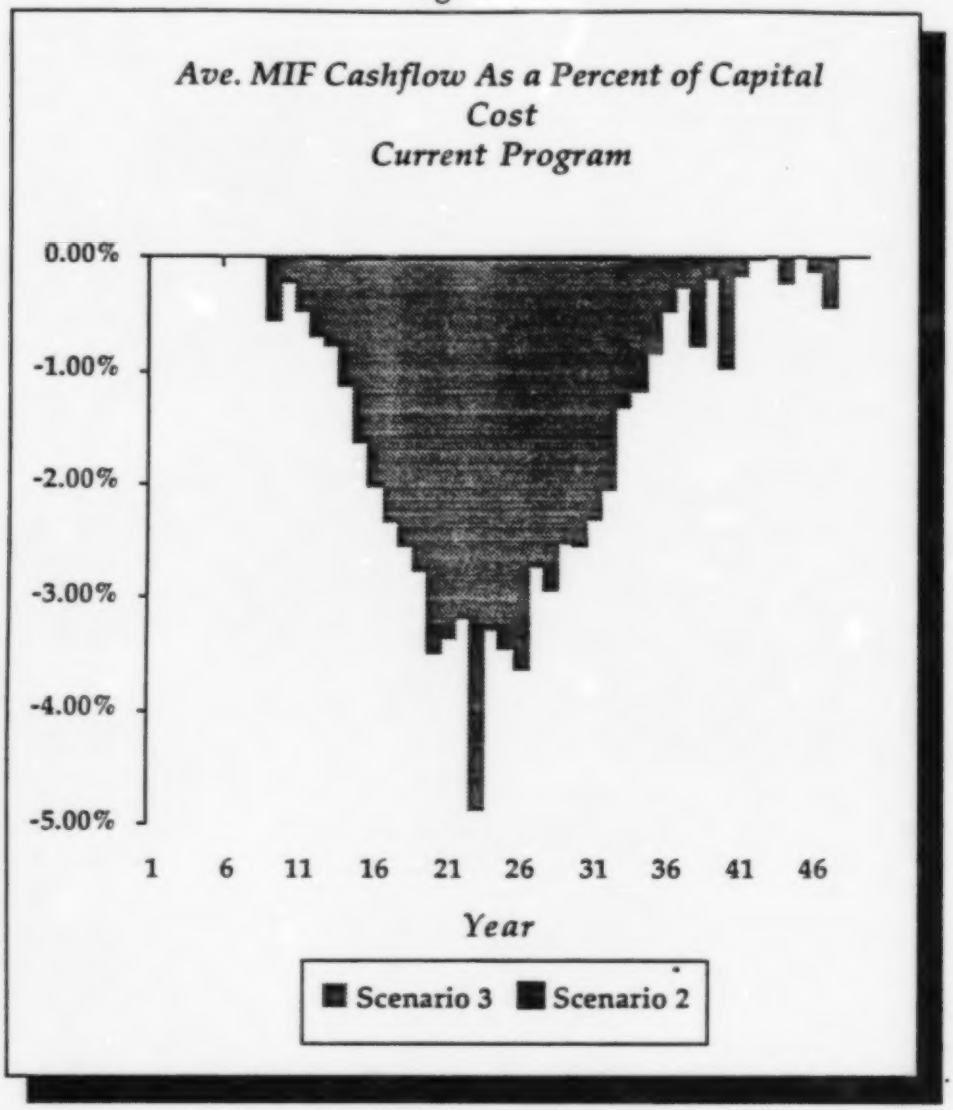

Ave. MIF Cashflow As a Percent of Capital Cost
Current Program
0.00%
-1.00%
-2.00%
-3.00%
-4.00%
-5.00%
1
6
11
16
21
26
31
36
41
46
Year
Scenario 3
Scenario 2

Figure VI.8

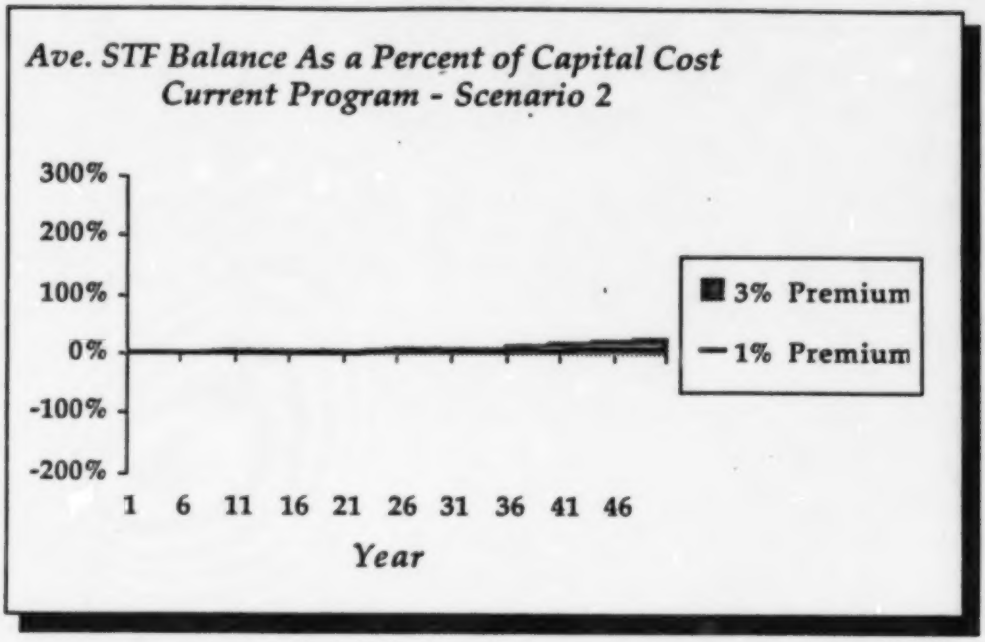


Figure VI.9

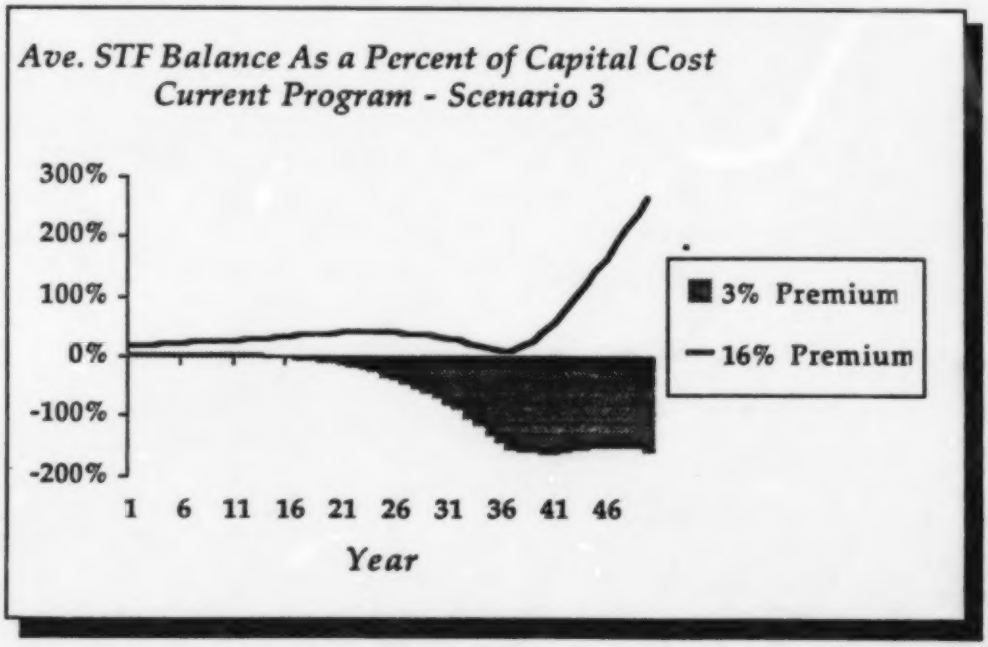
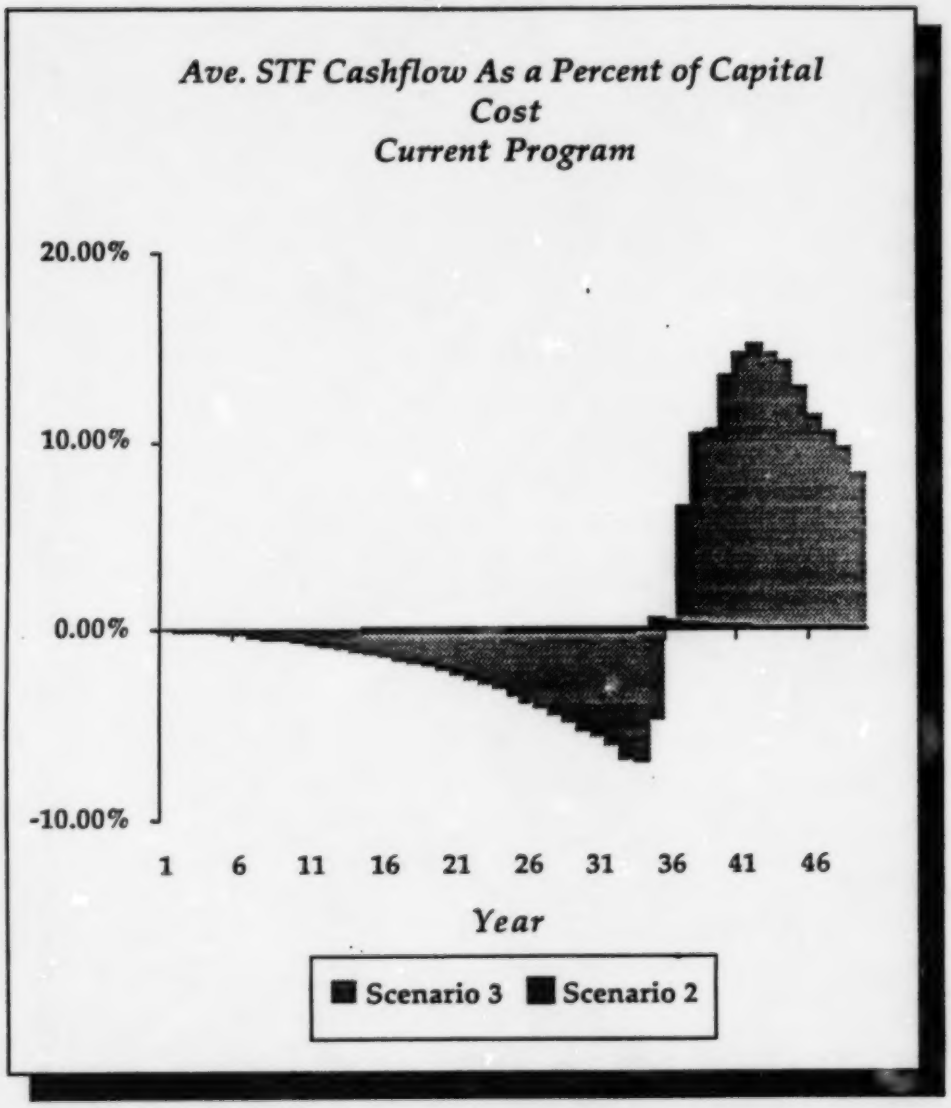


Figure VI.10


Ave. STF Cashflow As a Percent of Capital Cost
Current Program
20.00%
10.00%
0.00%
-10.00%
1
6
11
16
21
26
31
36
41
46
Year
Scenario 3
Scenario 2

VII. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

The short history of the ILM in Canada means that any conclusions regarding its long term viability are tentative. However, based on the results of the study described in Section VI, the following conclusions can be made:

1. The level of "tilt" built into the program design directly affects the ILM default risk.
2. The level of defaults is heavily dependent on the long term difference between market rent increases and inflation. The study results indicate that defaults increase significantly when rent increases lag behind inflation by one percentage point.
3. The Current FCHP has the lowest overall default costs of the non-Cash-feed designs examined in this study.
4. Program designs which provide for unlimited federal assistance ("Cash-feed" programs) are more cost effective where rent increases lag inflation.
5. The ILM is generally more cost effective than an EPM since the EPM requires more up-front assistance.
6. The Cash-feed ILM program design is the most cost effective overall. It is also the least sensitive to changing economic conditions.
7. The present 3% MIF and STF premiums are higher than necessary if long term increases in rents equal or exceed long term inflation (assuming that adequate provision has been made for financial problems due to poor management).
8. Supporting the Current FCHP with a taxable STF is costly. STF premiums would be lower if the STF was tax-exempt.

B. *Recommendations*

The following recommendations regarding program design should be considered as part of the review of the Current FCHP:

1. If an ILM is to be part of the program design, the tilt factor should be retained in the program. Further study is needed to analyze the optimal level of tilt.

2. While the Cash-feed ILM program design was found to be the most cost effective, it does not account for any loss of incentive for co-ops to minimize operating costs. If a Cash-feed program were implemented without a ceiling on federal assistance, then the subsequent increase in operating costs may significantly reduce the cost effectiveness of such a program. Cash-feeding, however, may be an effective alternative to the present STF for assisting projects in financial difficulty.

3. If the STF is to be continued, every effort should be made to secure its tax-exempt status. Otherwise, assistance for projects experiencing difficulties should be funded through another tax-exempt vehicle.

4. If the current program design is retained, further analysis should be conducted to determine appropriate premium levels. There may be some justification for lower premiums if it can be shown that the worst-case (scenario 3) is not likely to occur.

APPENDICES

APPENDIX A - MODELLING ECONOMIC CONDITIONS

This appendix describes the methods used to derive the models for each economic variable. This will be accomplished by presenting the analysis and results which lead to the determination of a model for the prime lending rate.

A. Economic Distributions

To create a simulated distribution of an economic variable, Y, there must be a function, f, which will transform one or more input variables into a unique value of Y. The input variables may include:

1. current or previously observed values of variables which are independent of the variable Y; or
2. previously observed values of the variable Y,

and usually include a variable, ε, which introduces random deviations from the expected values of Y. Thus, at any time t , the observed value of the economic variable, Y_t, may be determined from the formula:

$$Y_t = f(X_t, \varepsilon_t),$$

where X_t is a vector of input variables measured at time t. The distribution of Y_t will have a variance which is determined by the variance of ε_t and an expected value which is determined by the values of the input variables X_t (assuming that the expected value of ε_t is zero.)

To create a realistic model for the variable Y, it is necessary to first determine which variables can be used to explain the variability in Y. The list of candidate variables need not be restricted to those variables which have a strict causal effect on Y. There may be some variables which do not have a causal effect on the outcome of Y, but do help to explain the variability in Y. As an example, the analysis for this study determined that local inflation rates may be used to help explain the variability in local market rent increases. However, it is not clear that local inflation rates have a causal effect on local market rent increases. Indeed, the instrument used to measure the inflation rate (Consumer Price Index) contains a component for market rent increases, implying that the opposite may be more likely.

Once the significant variables have been identified, the form of the function f may be estimated, as well as the distribution of the random variable ε. The following subsections describe the methodology used to determine a model for the prime lending rate.

B. *Modelling the Prime Lending Rate*

1. Data

Data collected included annual values of the prime lending rate and the Consumer Price Index (CPI) as measured at the end of October for the period 1969 to 1989 inclusive. The prime rate equals or exceeds the inflation rate in all but 3 years during that period.

Since the inflation rate and the real prime rate were considered to be variables which would affect the nominal prime rate, it is first necessary to determine a model for the real prime rate. The difference between the nominal prime rate and the CPI is the real prime rate. The nominal prime rate at time t is labelled P_t, the rate of increase in the CPI at time t is labelled I_t, and the real prime rate is labelled S_t. The following relationship defines the real prime rate as a function of the nominal prime rate and the CPI:

$$S_t = \frac{1 + P_t}{1 + I_t} - 1.$$

The real prime rate was calculated for the period 1969 to 1989 inclusive.

2. Transformations

Linear regression is used to determine the model for the real prime rate. There are several conditions which must be satisfied when using this analysis. Two important conditions are:

a. that the equation to be estimated must be linear in its parameters; and

b. that the variance of the response variable be uniform over all values of the response variable.

The second assumption will likely be violated by any variable that is a proportion, since the variance tends to increase as the value of the observations increase. Thus, a transformation must be made to the response variable, S_t, to satisfy these conditions. The most common transformation performed on response variables which are expressed as proportions is the natural logarithm of 1 + the response variable. Therefore, the transformation is:

$$\ln(1+S_t) = \ln(1+P_t) - \ln(1+I_t).$$

3. **Identification of Significant Factors**

In the analysis of time series it is usually the case that the value of a variable at time t is somewhat dependent on the value of that variable in the previous time period (t-1). In the case of the real prime rate this seems justified, since an investors expectations in a year may be affected by his knowledge of real prime rates. In fact, a test of the partial autocorrelation functions for the real prime rate indicated that the series $\ln(1+S_t)$ is autoregressive process of order one, AR(1). Therefore, the logarithm of the previous period's real prime rate should be used to help estimate the current year's rate.

The other variable which was thought to affect the real prime rate at time t was the inflation rate. Since this variable was also found to be autoregressive of order one (see Section III), both the current year's value of the logarithm of inflation, $\ln(1+I_t)$, and the previous year's value, $\ln(1+I_{t-1})$, were considered to be candidate variables.

4. **Estimation and Model Building**

The general form of the equation to be estimated for the real prime rate at time t is:

$$\ln(1+S_t) = \mu + \beta_1\ln(1+S_{t-1}) + \beta_2\ln(1+I_t) + \beta_3\ln(1+I_{t-1}) + \varepsilon_t,$$

where, ε_t is an independent and identically distributed normal random variable with mean 0 and variance σ^2.

Linear regression was used to estimate the parameters of the equation. The estimates of the parameters are presented in the following table:

Parameter	Estimate	Standard Error	P-value*
μ	0.0113	0.0174	0.524
β_1	0.7178	0.1769	0.001
β_2	-0.6336	0.2026	0.007
β_3	0.6423	0.1984	0.005

* The p-value represents the probability of erroneously declaring a parameter to be different from zero when, in fact, the true parameter value is equal to zero. Thus, high p-values (e.g. greater than say .05) indicate that the hypothesis that the parameter value is equal to zero cannot be rejected.

Note that the estimate of the constant, μ, is very close to zero. In fact, a test of the hypothesis that the constant is zero results in a p-value of 0.524 which indicates that there is not enough data to support the inclusion of μ in the equation. Thus, it was concluded that this parameter should be deleted from the equation. The other parameters, on the other hand, indicate that the respective variables explain a significant portion of the variability in the real prime rate. The regression equation is significant, with an overall p-value of 0.0001, and the proportion of the variability of S_t explained by the independent variables is 65.8% (as measured by the adjusted R^2).

If the constant term is deleted then the equation becomes:

$$\ln(1+S_t) = \beta_1 \ln(1+S_{t-1}) + \beta_2 \ln(1+I_t) + \beta_3 \ln(1+I_{t-1}) + \varepsilon_t$$

The equation is re-estimated and the subsequent estimates are:

Parameter	Estimate	Standard Error	P-value
β_1	0.8081	0.1079	0.000
β_2	-0.5726	0.1766	0.005
β_3	0.6960	0.1774	0.001

The p-values for the three parameters are all more significant than the in the previous equation. Furthermore, the adjusted R^2 of the second equation is higher at 86.4%, while the overall p-value is less at 0.000. The standard deviation of the error parameter is 1.79%.

Therefore, the equation which best fits the logarithm of the real rate of return data is

$$\ln(1+S_t) = (0.8081)\ln(1+S_{t-1}) - (0.5726)\ln(1+I_t) + (0.6960)\ln(1+I_{t-1}).$$

Taking the exponential on both sides of the equation gives

$$1+S_t = (1+S_{t-1})^{0.8081} \times (1+I_t)^{-.5726} \times (1+I_{t-1})^{0.696}.$$

Substituting 1+St with

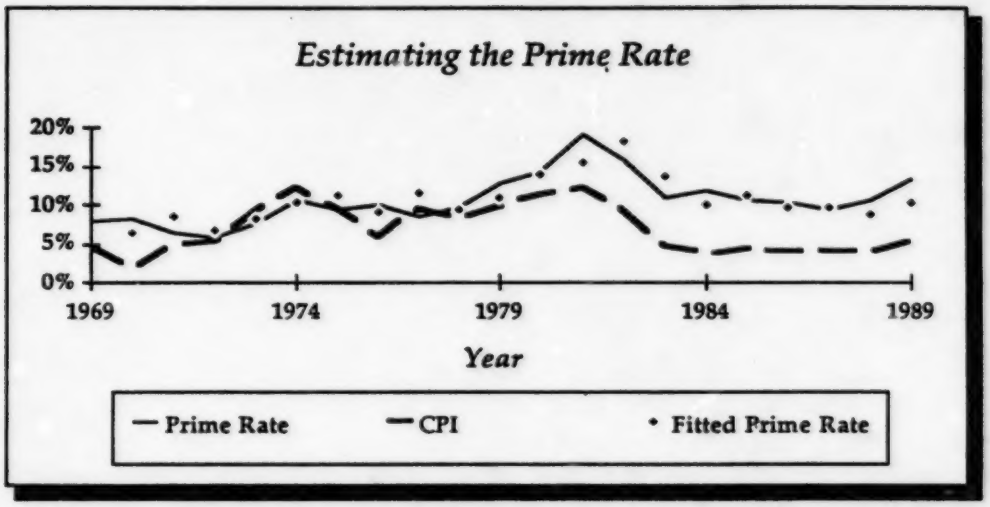
$$1+S_t = \frac{1+P_t}{1+I_t},$$

and solving for P_t gives

$$P_t = (1+P_{t-1})^{.80812}(1+I_t)^{.42744}(1+I_{t-1})^{-.1122} - 1.$$

The forecast of the prime rate at any time t is a weighted geometric average of the prime rate at time t-1, the inflation rate at time t, and the inflation rate at time t-1. Figure A.1 shows the forecast value of the prime rate for the years 1970 to 1989.

Figure A.1 - Estimating the Annual Prime Rate



C. *Estimation Results*

The results of parameter estimation of other economic variables are presented below. Similar methods as those described above were used to estimate the model parameters. These results were used to develop the final models presented in Section V above. In all cases the standard deviation of the error term may be found in Section V.

1. Vacancy

a. Model:

$$\ln(V_t) = \beta_0 + \beta_{1i} + \beta_2 \ln(V_{t-1}) + \beta_3 \ln(V_{t-1}) + \beta_4 \ln(V_{t-1}) + \varepsilon_t$$

where, V_t is the vacancy rate at time t.

b. Parameter Estimates

Parameter	Estimate	Standard Error	P-value
β_0	0.1193	0.0434	0.000
β_{11}	-0.2322	0.1118	0.000
β_{12}	0.0593	0.1064	0.000
β_{13}	0.0510	0.0941	0.000
β_{14}	-0.0705	0.0615	0.000
β_{15}	0.1438	0.0853	0.000
β_{16}	0.1020	0.0921	0.000
β_2	0.7066	0.0569	0.000
β_3	0.4959	0.0626	0.000
β_4	-0.4437	0.0556	0.000

2. National Inflation Rate

a. Model:

$$\ln(1+I_t) = \beta_0 + \beta_1 \ln(1+I_{t-1}) + \varepsilon_t$$

where, I_t is the annual percentage increase in the national CPI for the year ending at time t.

b. Parameter Estimates

Parameter	Estimate	Standard Error	P-value
β_0	0.0143	0.0105	0.191
β_1	0.7858	0.1455	0.000

3. **Local Inflation Rate**

a. Model:

$$\ln(1+L_t) = \beta_1 \ln(1+I_t) + \varepsilon_t$$

where, L_t is the annual percentage increase in the regional CPI for the year ending at time t.

b. Parameter Estimates

Parameter	Estimate	Standard Error	P-value
β_1	0.9750	0.0122	0.000

4. **Local Rent Increases**

a. Model:

$$\ln(1+R_t) = \beta_0 + \ln(1+I_t) + \varepsilon_t$$

where, R_t is the annual percentage increase in the regional market rent for the year ending at time t.

b. Parameter Estimates

Parameter	Estimate	Standard Error	P-value
β_0	0.8332	0.0533	0.000

5. **Mortgage Rate**

a. Model

$$\ln(1+m_t) = \beta_1 \ln(1+m_{t-1}) + \beta_2 \ln(1+I_t) +$$

$$\beta_3 \ln(1+I_{t-1}) + \beta_4 \ln(1+P_t) + \varepsilon_t$$

where, the real rate mortgage m_t is defined to be

$$m_t = \frac{1 + M_t}{1 + I_t} - 1.$$

M_t is the nominal mortgage rate at time t.

b. Parameter Estimates

Parameter	Estimate	Standard Error	P-value
β_1	0.4895	0.0642	0.000
β_2	-0.9569	0.0754	0.000
β_3	0.5130	0.0924	0.000
β_4	0.5302	0.0831	0.000

APPENDIX B - PROJECT DATA

A. *Data Adjustments*

The data provided was not always internally consistent.

The mortgage payment and capital cost data were inconsistent for some projects. For these projects, the capital cost of the project was recalculated (by working back from the mortgage payment and real interest rate) to ensure that it would be consistent with the mortgage payment coded on the data file. If the capital cost was recalculated, the replacement reserve contribution was also recalculated.

The following formula was used for recalculating the replacement reserve contribution:

replacement reserve contribution = 0.65% x 70% x revised capital cost

Many projects' first year cashflow was either below or above the break-even level.

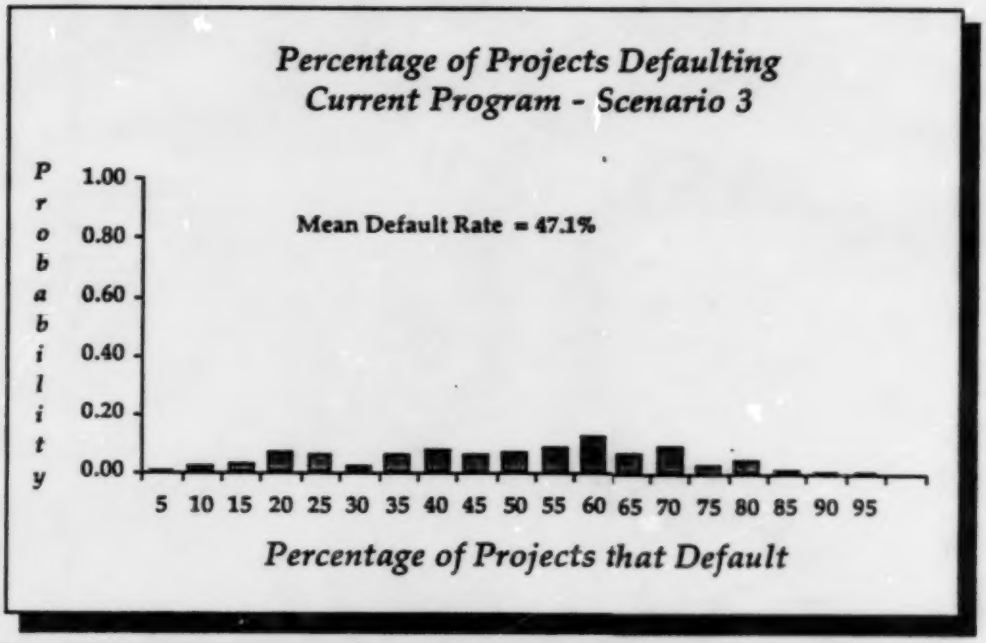
Each project should have a net projected cashflow of zero in the first year. This means that occupancy charges plus federal assistance should equal total operating costs including the mortgage payment. For those co-ops where there was a discrepancy, the operating costs were re-calculated so that the cashflow would balance.

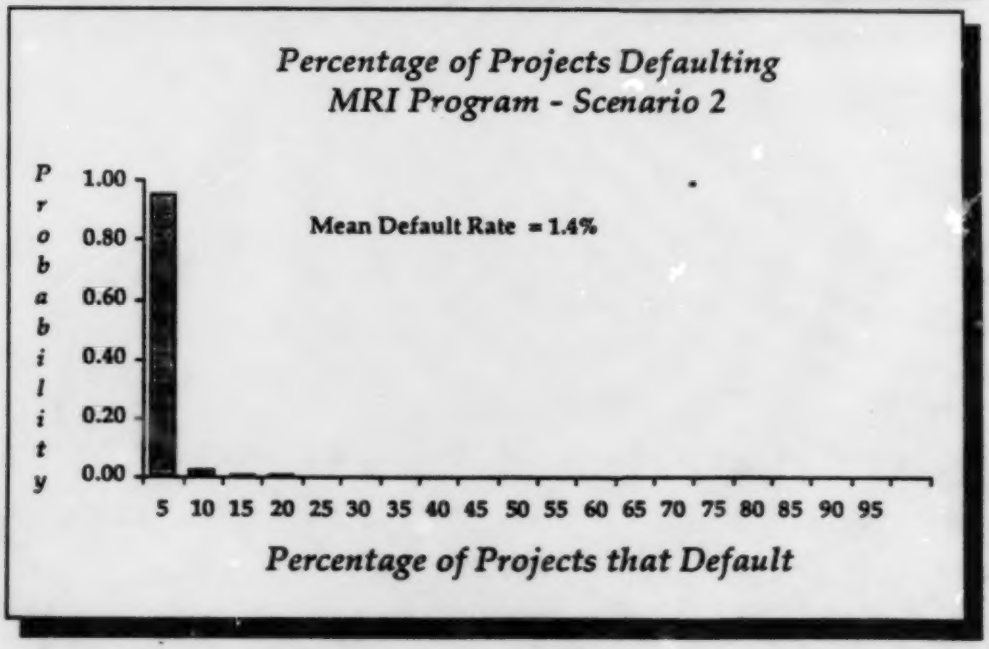
The following formula was used to re-calculate operating costs:

operating costs = occupancy charge + federal assistance
- mortgage payment
- replacement reserve contribution
- security of tenure fund contribution

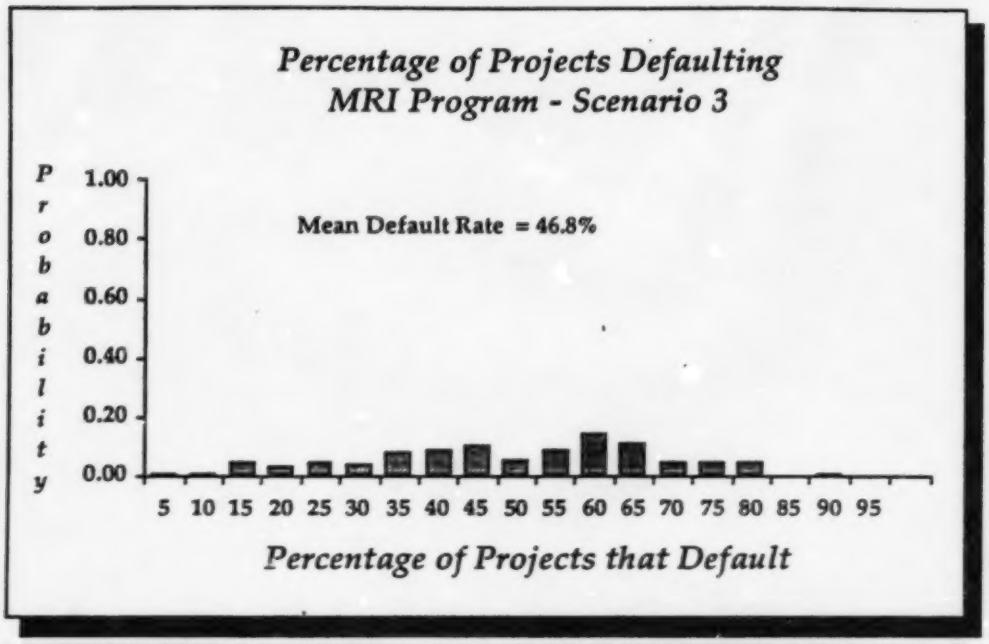
APPENDIX C - FIGURES

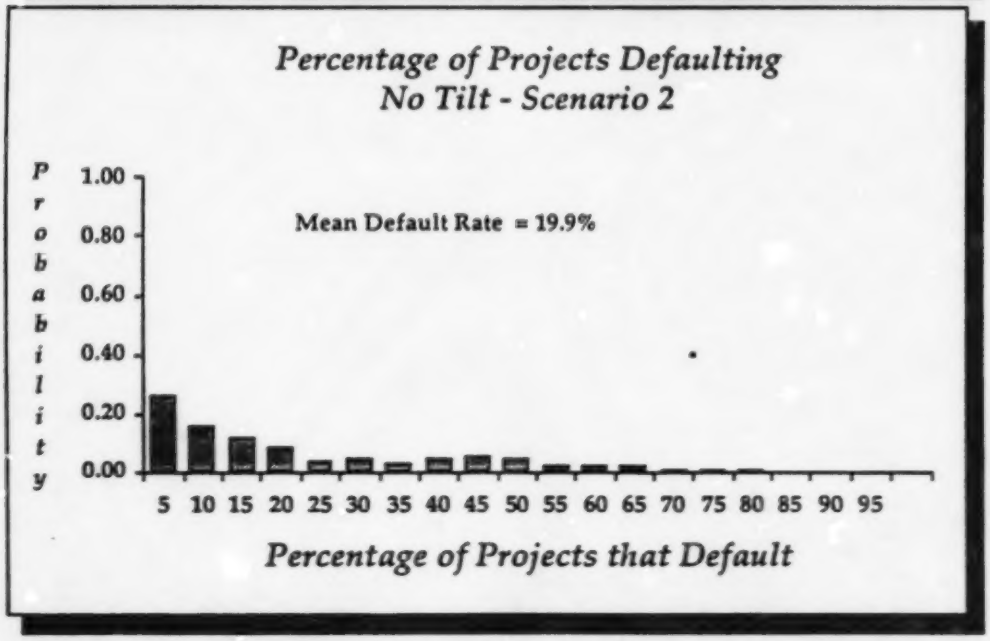
The following charts represent the distributions described in Section VI for the three non-Cash-feed programs under scenarios 2 and 3.



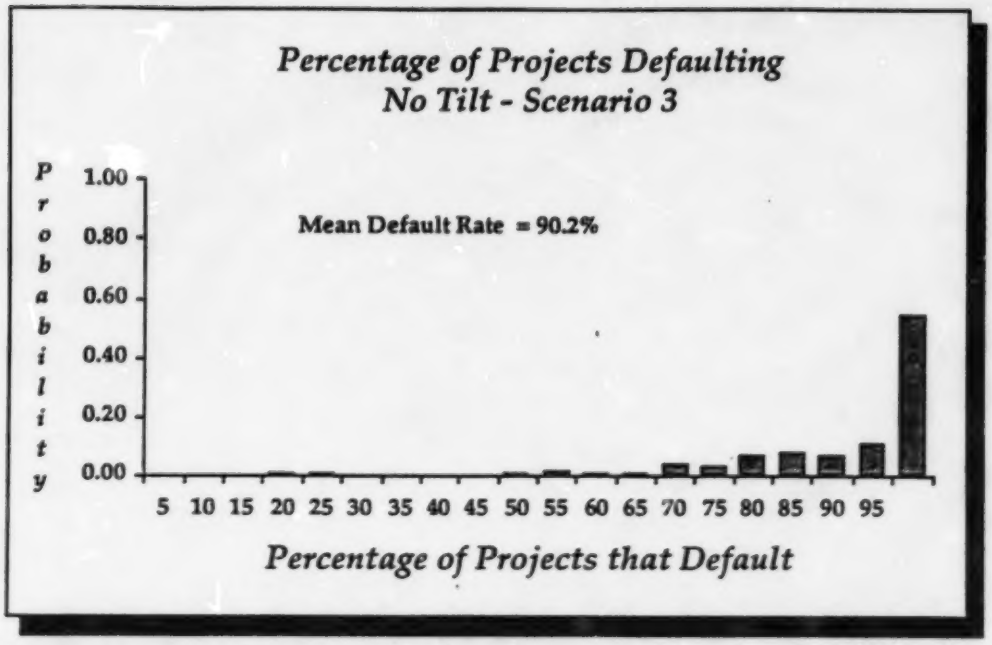


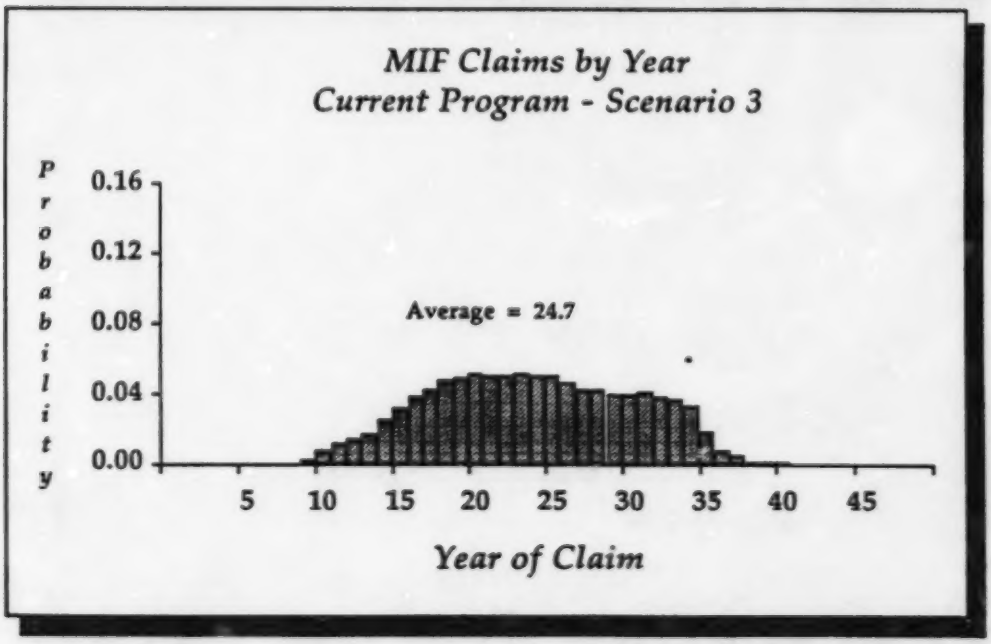
Each bar on the above graphs represents the probability that the percentage of projects defaulting lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that between 10.01% and 15.00% inclusive of projects default is represented by the bar labelled 15.



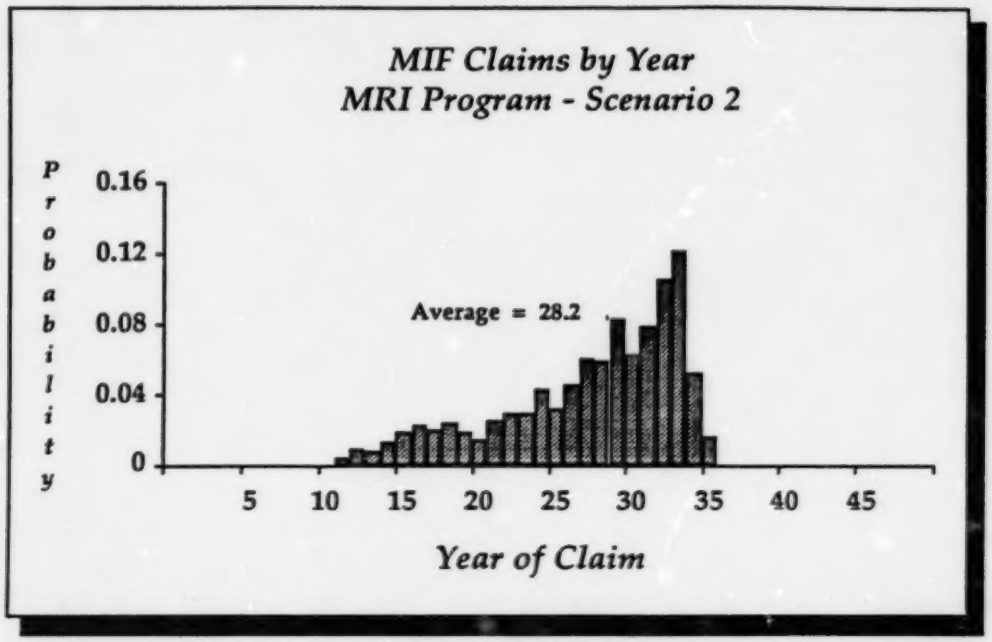


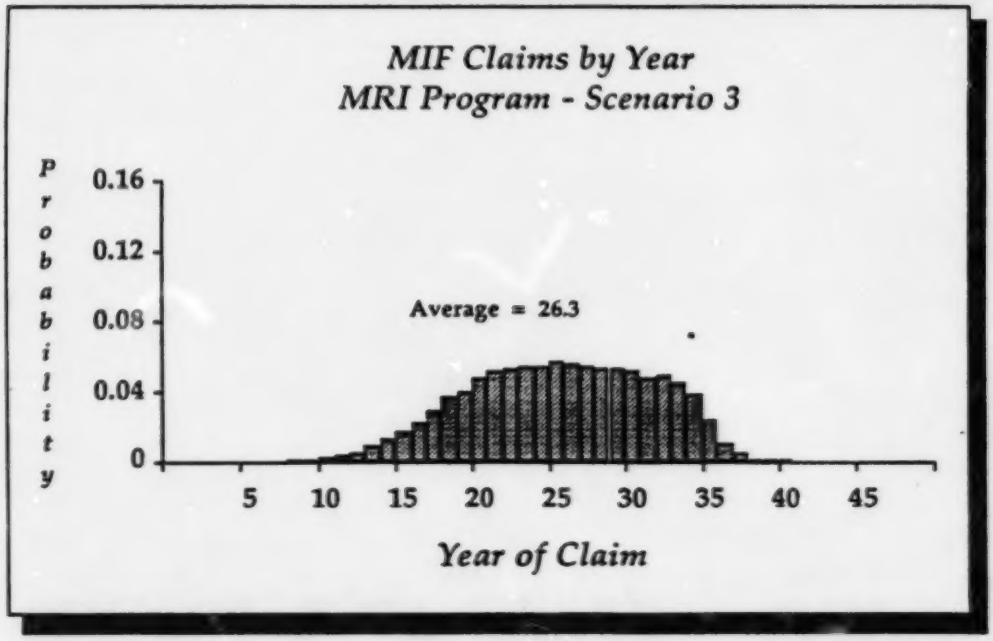
Each bar on the Percentage of Projects Defaulting graphs represents the probability that the percentage of projects defaulting lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that between 10.01% and 15.00% inclusive of projects default is represented by the bar labelled 15.

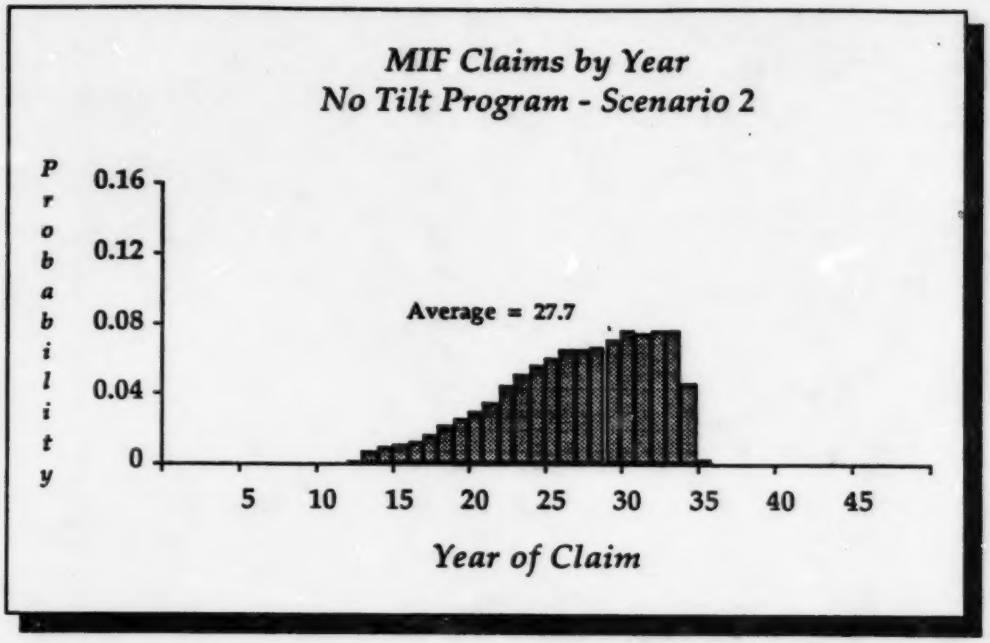


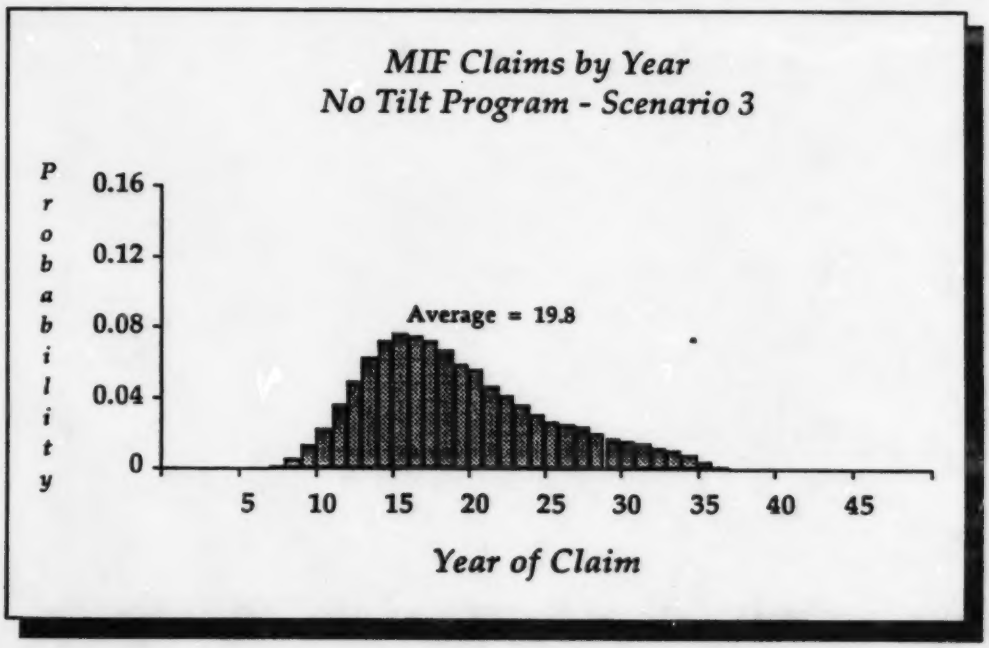


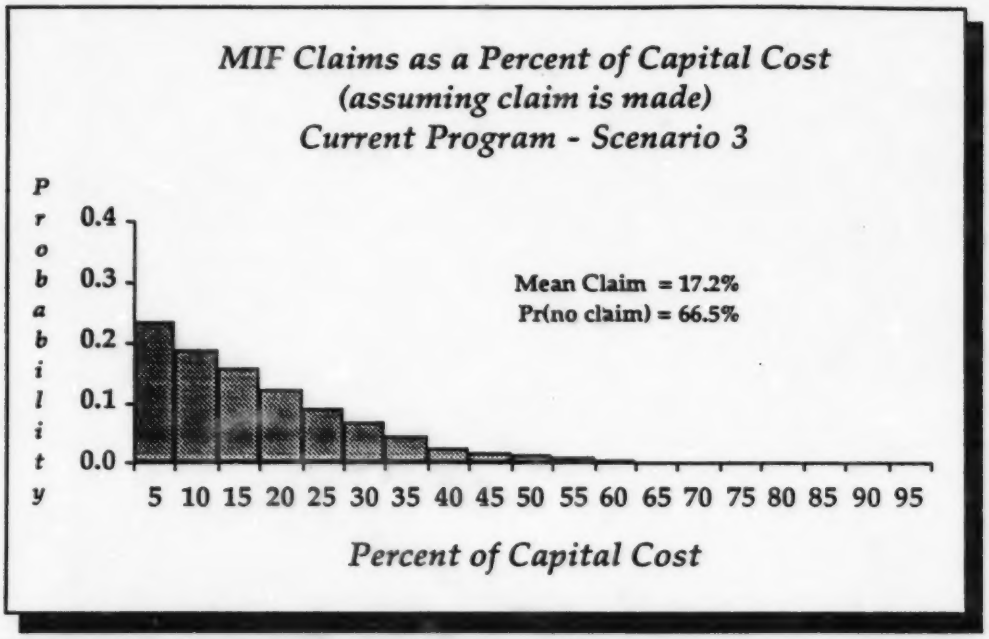
Each bar on the Percentage of Projects Defaulting graphs represents the probability that the percentage of projects defaulting lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that between 10.01% and 15.00% inclusive of projects default is represented by the bar labelled 15.

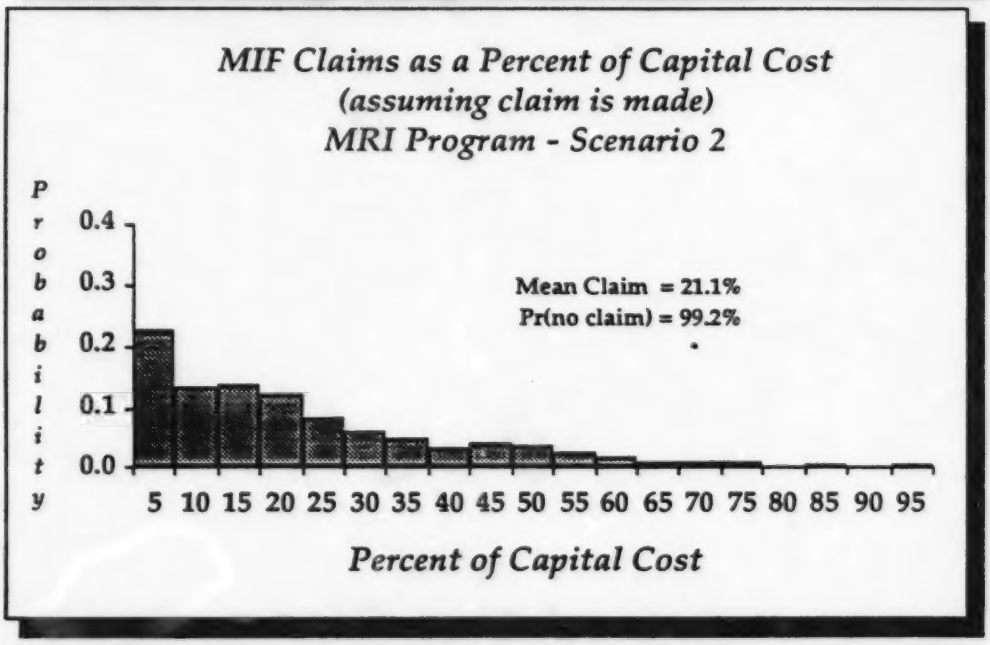

MIF Claims by Year
MRI Program - Scenario 2
Probability
0.16
0.12
0.08
0.04
0
Average = 28.2
5
10
15
20
25
30
35
40
45
Year of Claim


MIF Claims by Year
MRI Program - Scenario 3
Probability
0.16
0.12
0.08
0.04
0
Average = 26.3
5
10
15
20
25
30
35
40
45
Year of Claim

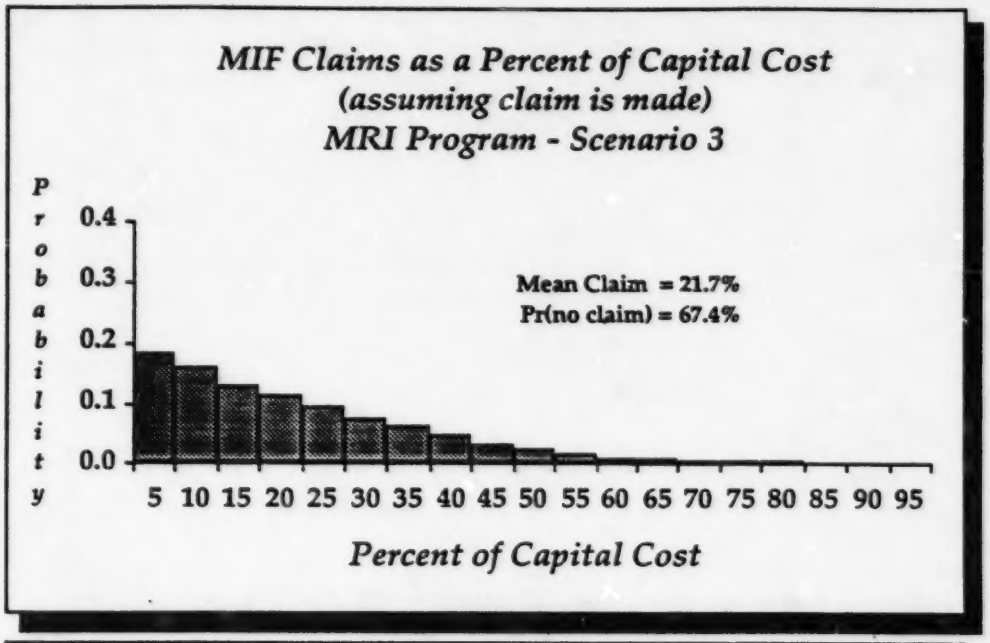

MIF Claims by Year
No Tilt Program - Scenario 2
Probability
0.16
0.12
0.08
0.04
0
Average = 27.7
5
10
15
20
25
30
35
40
45
Year of Claim

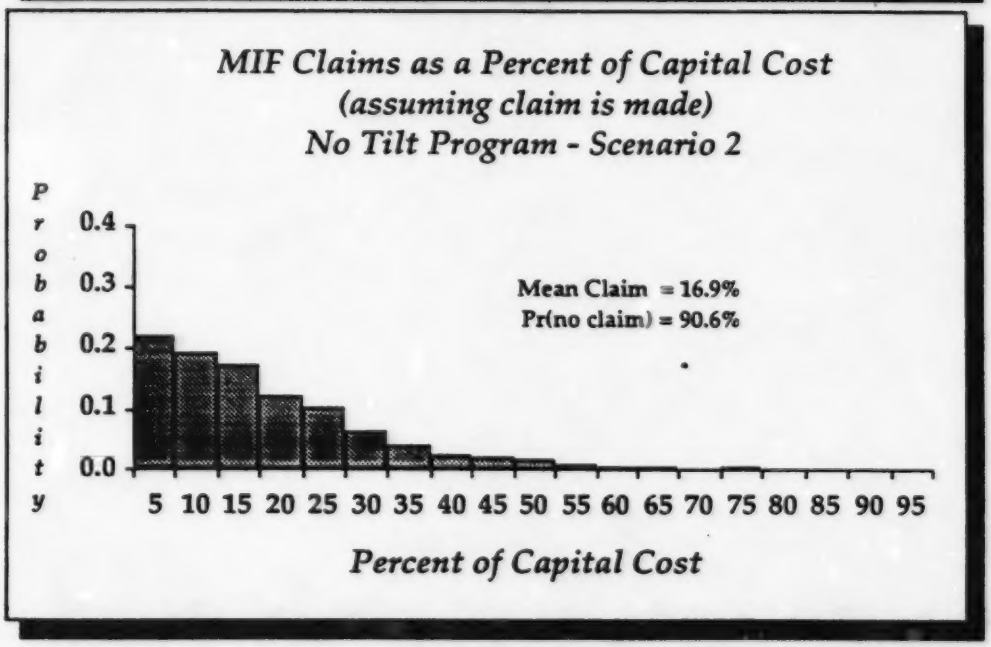

MIF Claims by Year
No Tilt Program - Scenario 3
Probability
0.16
0.12
0.08
0.04
0
Average = 19.8
5
10
15
20
25
30
35
40
45
Year of Claim


MIF Claims as a Percent of Capital Cost
(assuming claim is made)
Current Program - Scenario 3
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 17.2%
Pr(no claim) = 66.5%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

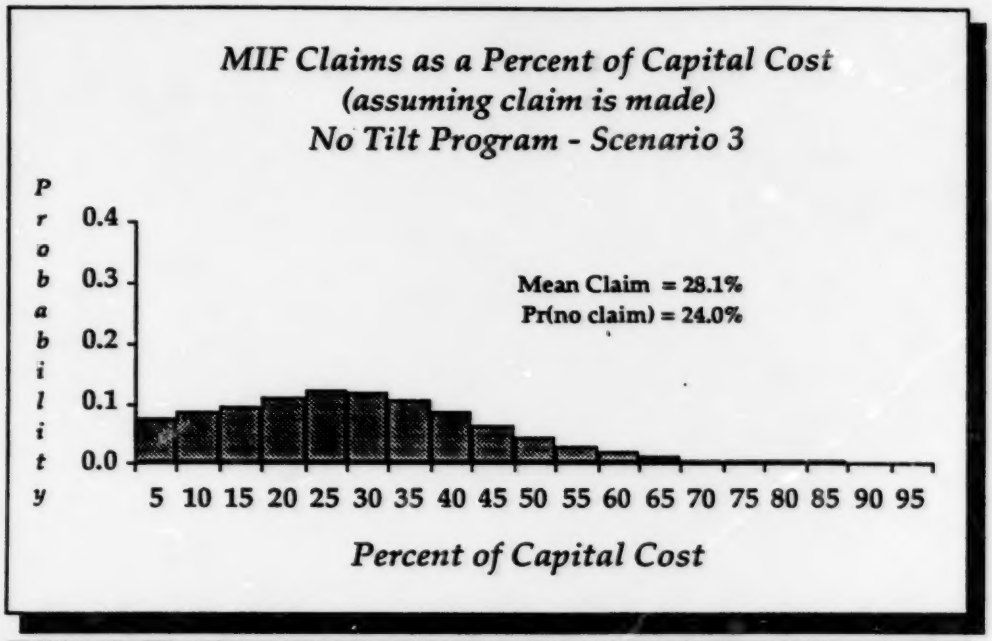

MIF Claims as a Percent of Capital Cost
(assuming claim is made)
MRI Program - Scenario 2
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 21.1%
Pr(no claim) = 99.2%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

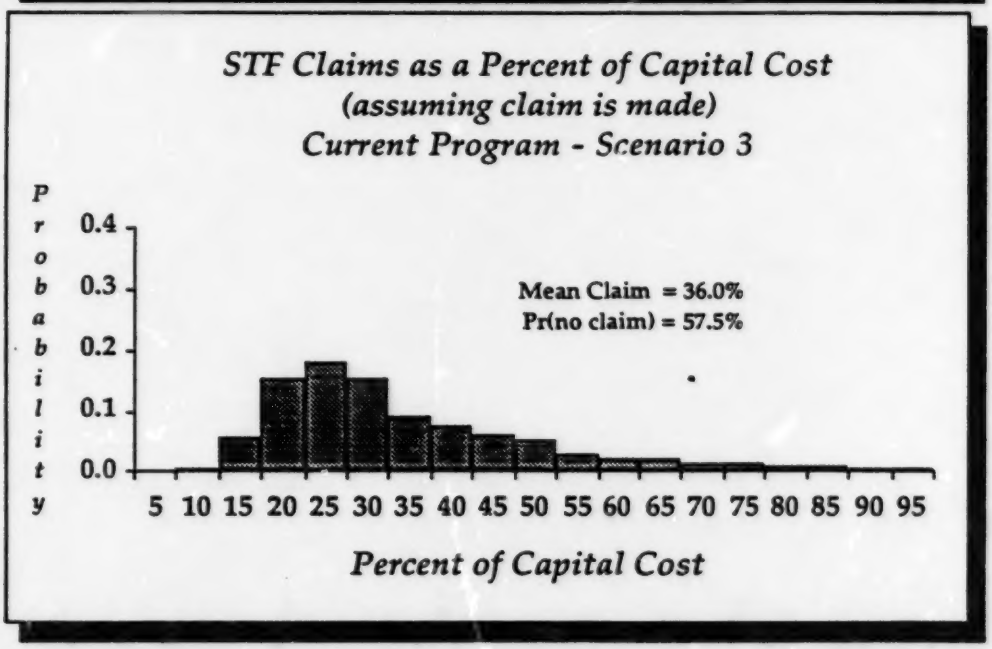
Each bar on the above graphs represents the probability that the claim, as a percentage of capital cost lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that the claim is between 10.01% and 15.00% of capital cost inclusive is represented by the bar labelled 15.


MIF Claims as a Percent of Capital Cost
(assuming claim is made)
MRI Program - Scenario 3
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 21.7%
Pr(no claim) = 67.4%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

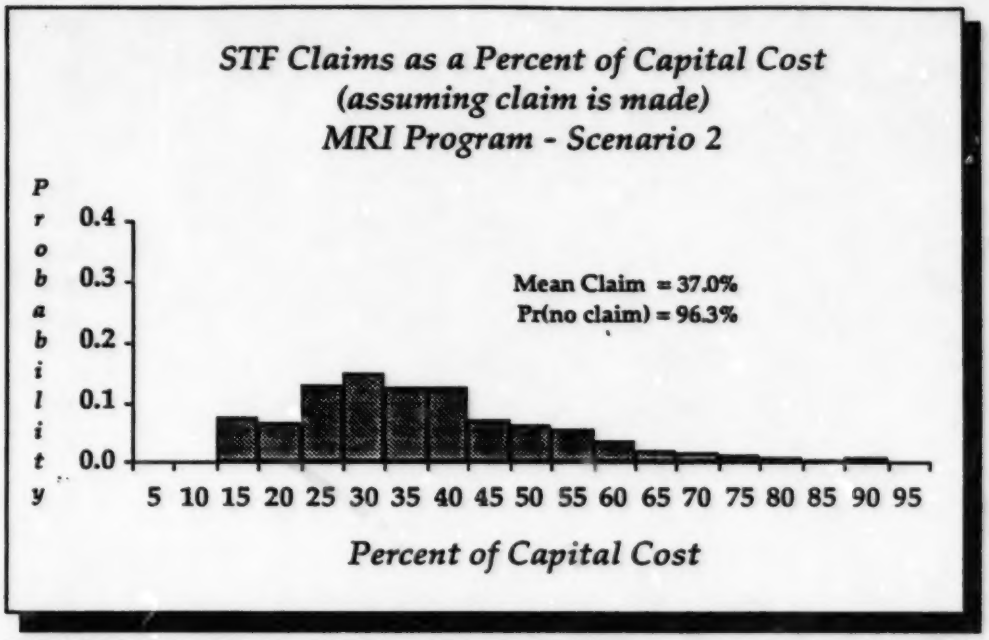

MIF Claims as a Percent of Capital Cost
(assuming claim is made)
No Tilt Program - Scenario 2
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 16.9%
Pr(no claim) = 90.6%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

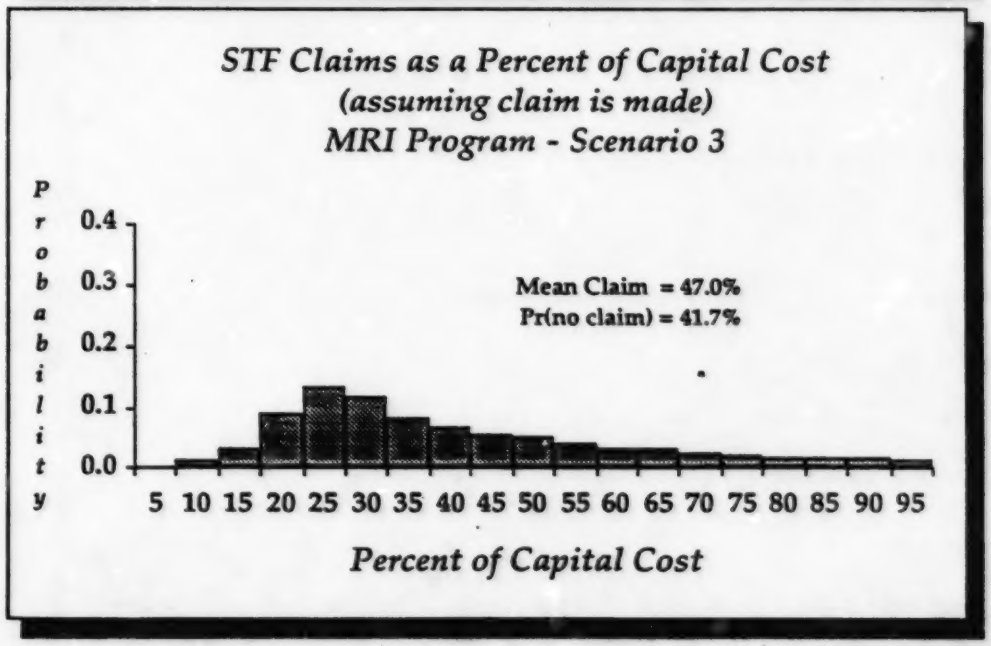
Each bar on the above graphs represents the probability that the claim, as a percentage of capital cost lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that the claim is between 10.01% and 15.00% of capital cost inclusive is represented by the bar labelled 15.


MIF Claims as a Percent of Capital Cost
(assuming claim is made)
No Tilt Program - Scenario 3
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 28.1%
Pr(no claim) = 24.0%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

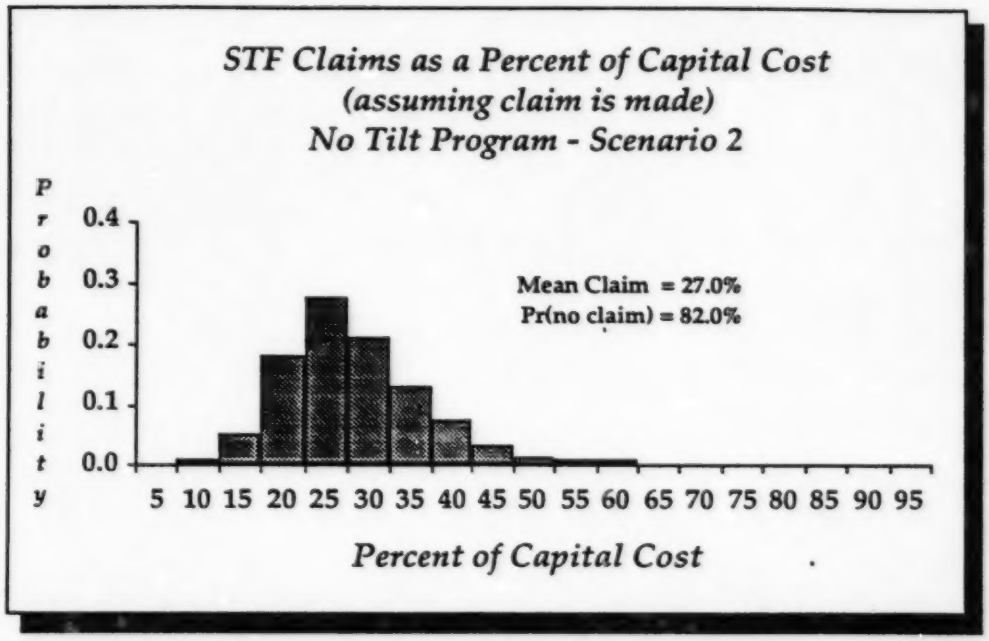

STF Claims as a Percent of Capital Cost
(assuming claim is made)
Current Program - Scenario 3
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 36.0%
Pr(no claim) = 57.5%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

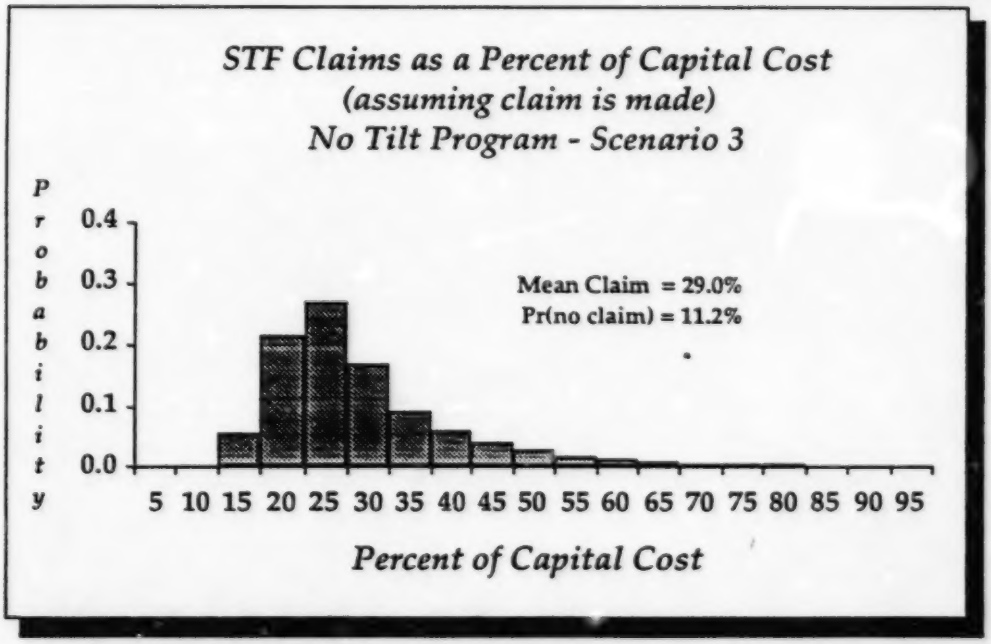
Each bar on the above graphs represents the probability that the claim, as a percentage of capital cost lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that the claim is between 10.01% and 15.00% of capital cost inclusive is represented by the bar labelled 15.


STF Claims as a Percent of Capital Cost
(assuming claim is made)
MRI Program - Scenario 2
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 37.0%
Pr(no claim) = 96.3%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

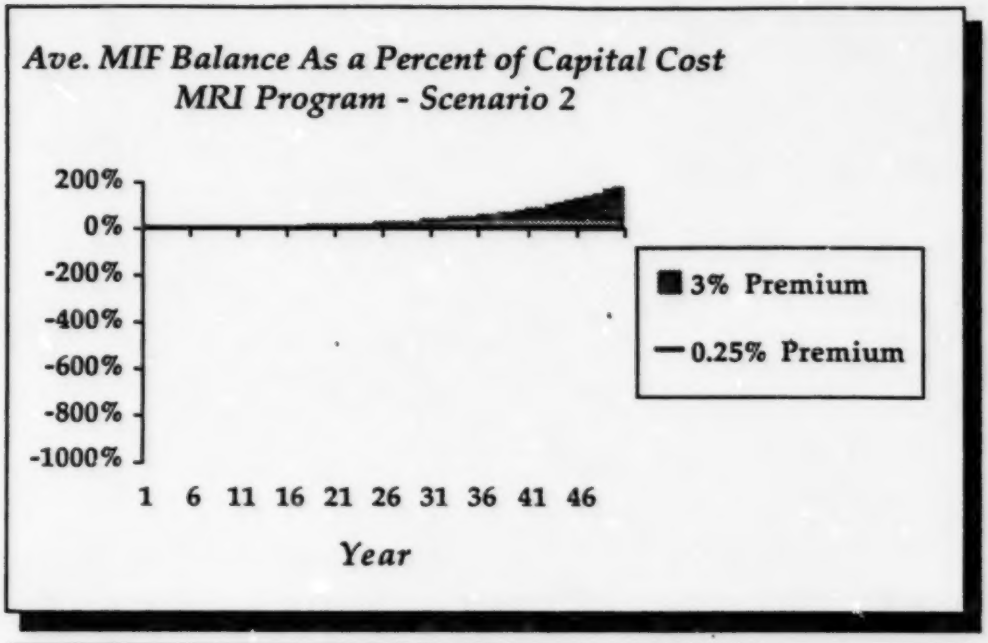

STF Claims as a Percent of Capital Cost
(assuming claim is made)
MRI Program - Scenario 3
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 47.0%
Pr(no claim) = 41.7%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

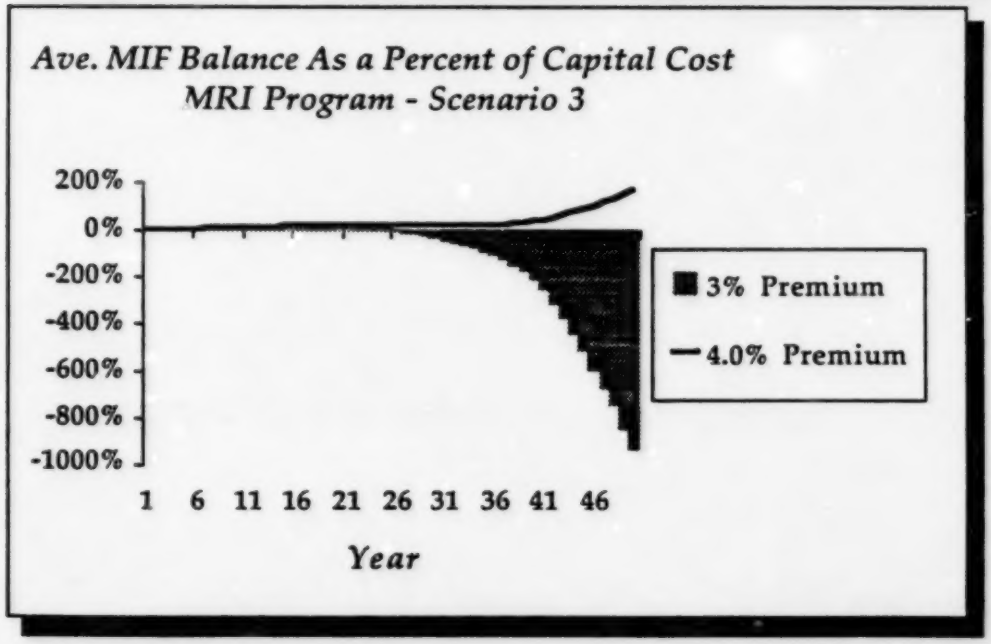
Each bar on the above graphs represents the probability that the claim, as a percentage of capital cost lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that the claim is between 10.01% and 15.00% of capital cost inclusive is represented by the bar labelled 15.

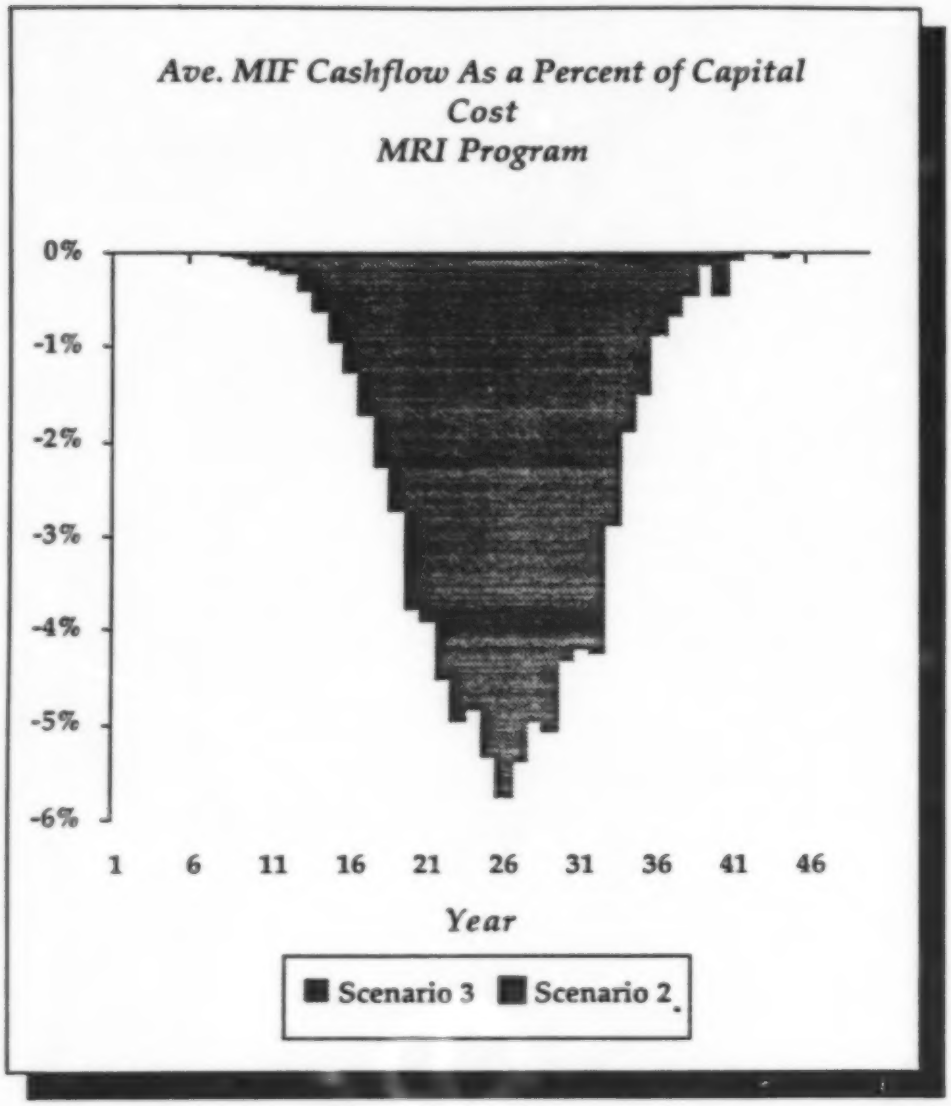

STF Claims as a Percent of Capital Cost
(assuming claim is made)
No Tilt Program - Scenario 2
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 27.0%
Pr(no claim) = 82.0%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost


STF Claims as a Percent of Capital Cost
(assuming claim is made)
No Tilt Program - Scenario 3
Probability
0.4
0.3
0.2
0.1
0.0
Mean Claim = 29.0%
Pr(no claim) = 11.2%
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Percent of Capital Cost

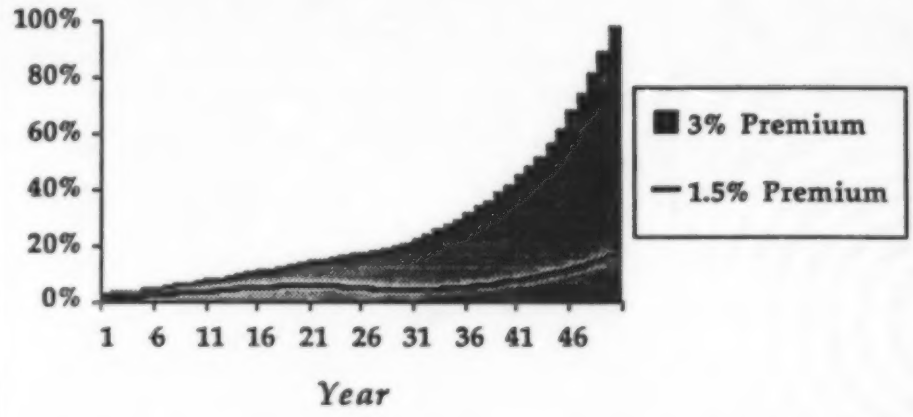
Each bar on the above graphs represents the probability that the claim, as a percentage of capital cost lies within a certain range. The labels under each bar indicate the upper end of the range. For example, the probability that the claim is between 10.01% and 15.00% of capital cost inclusive is represented by the bar labelled 15.


Ave. MIF Balance As a Percent of Capital Cost
MRI Program - Scenario 2
200%
0%
-200%
-400%
-600%
-800%
-1000%
1 6 11 16 21 26 31 36 41 46
Year
3% Premium
0.25% Premium

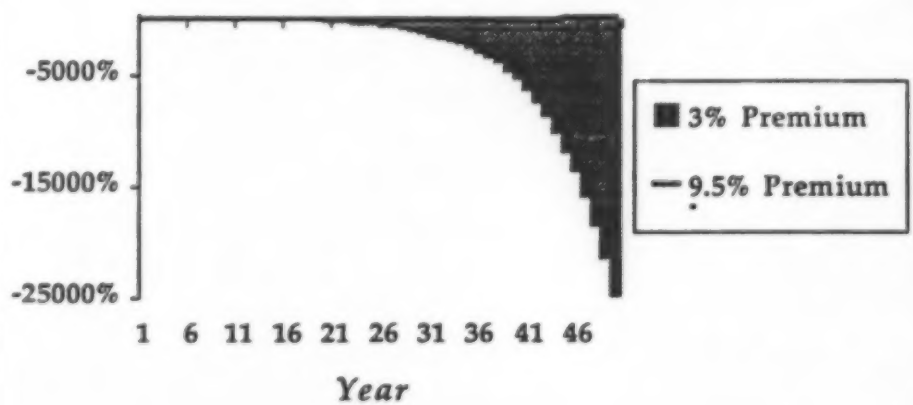

Ave. MIF Balance As a Percent of Capital Cost
MRI Program - Scenario 3
200%
0%
-200%
-400%
-600%
-800%
-1000%
1 6 11 16 21 26 31 36 41 46
Year
3% Premium
4.0% Premium

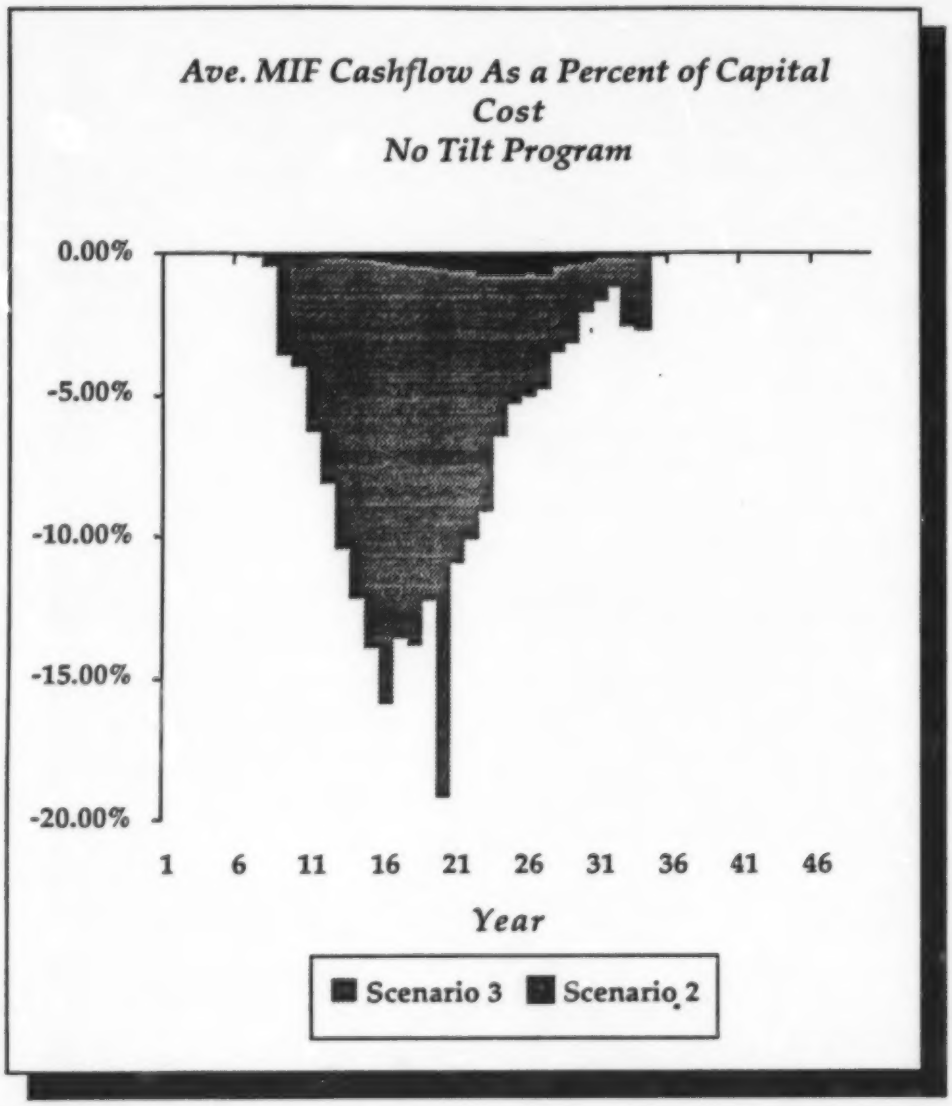

Ave. MIF Cashflow As a Percent of Capital Cost
MRI Program
0%
-1%
-2%
-3%
-4%
-5%
-6%
1
6
11
16
21
26
31
36
41
46
Year
Scenario 3
Scenario 2

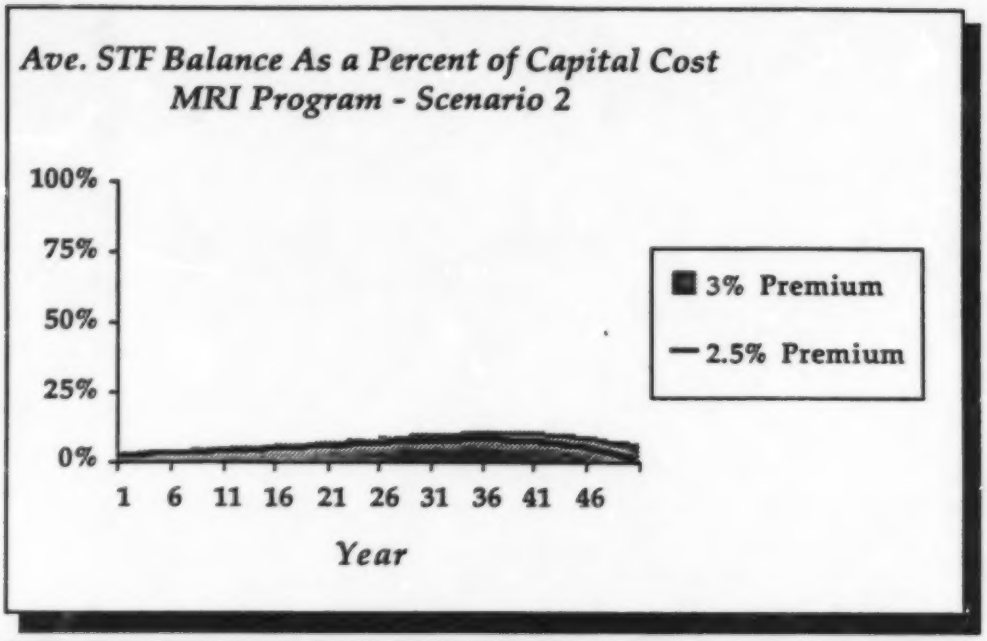
Ave. MIF Balance As a Percent of Capital Cost
No Tilt Program - Scenario 2

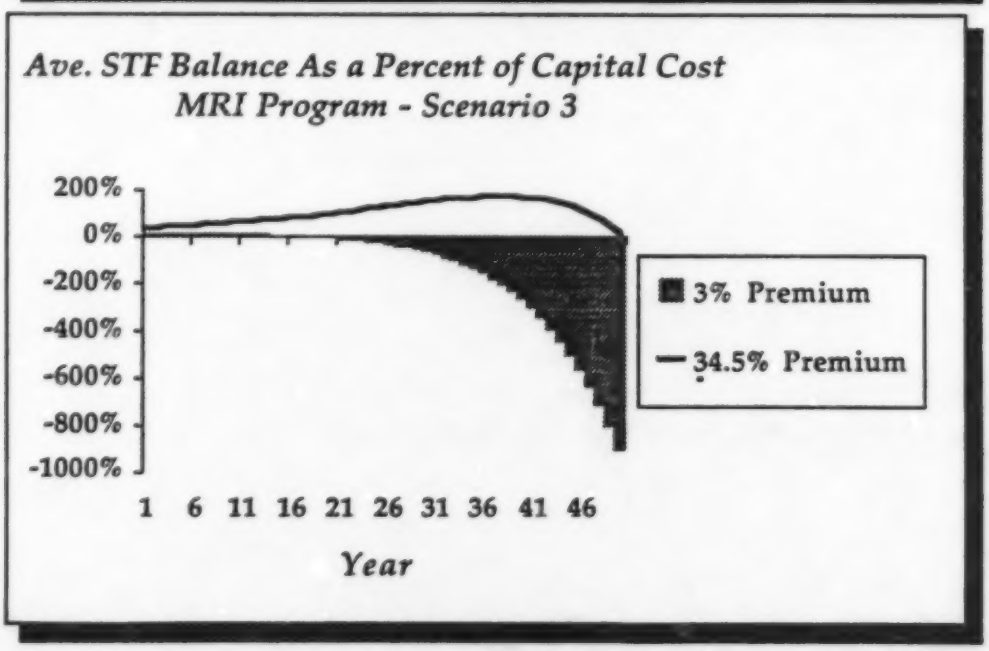


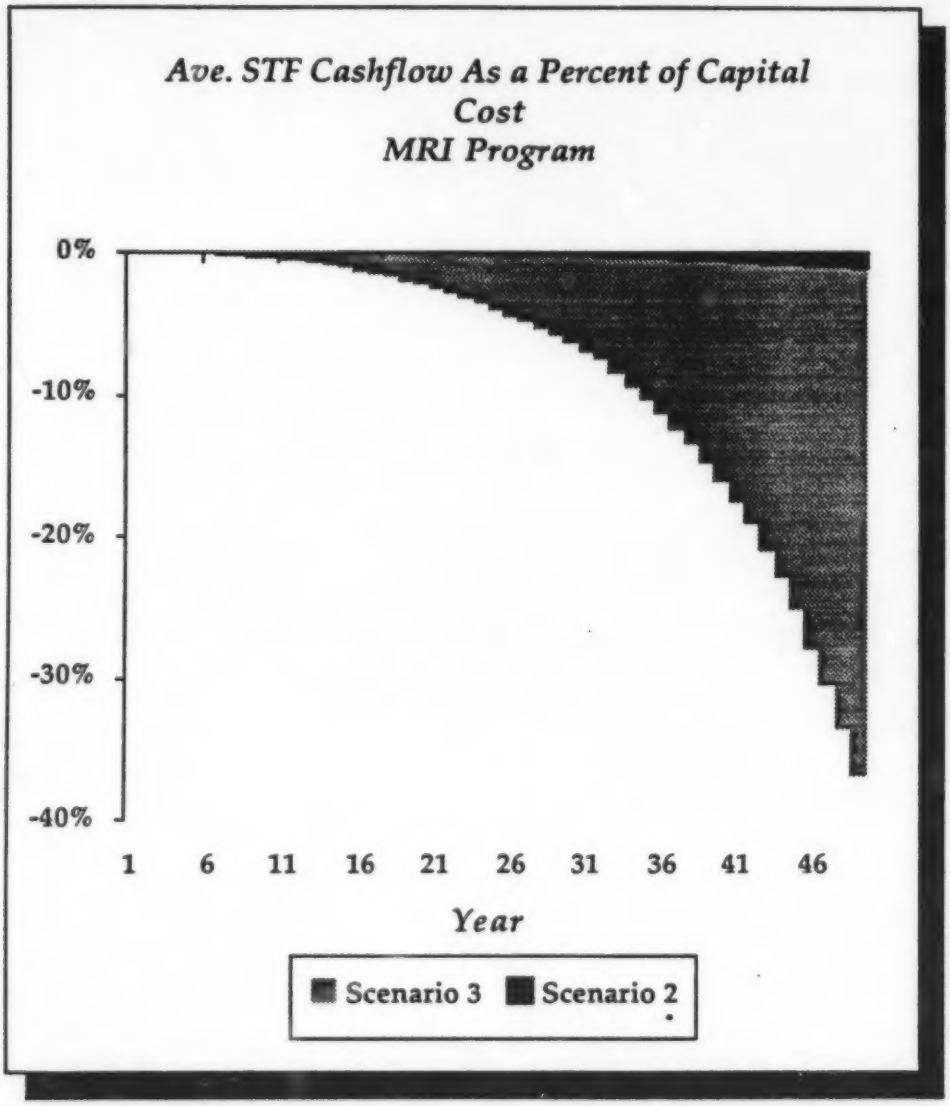
Ave. MIF Balance As a Percent of Capital Cost
No Tilt Program - Scenario 3

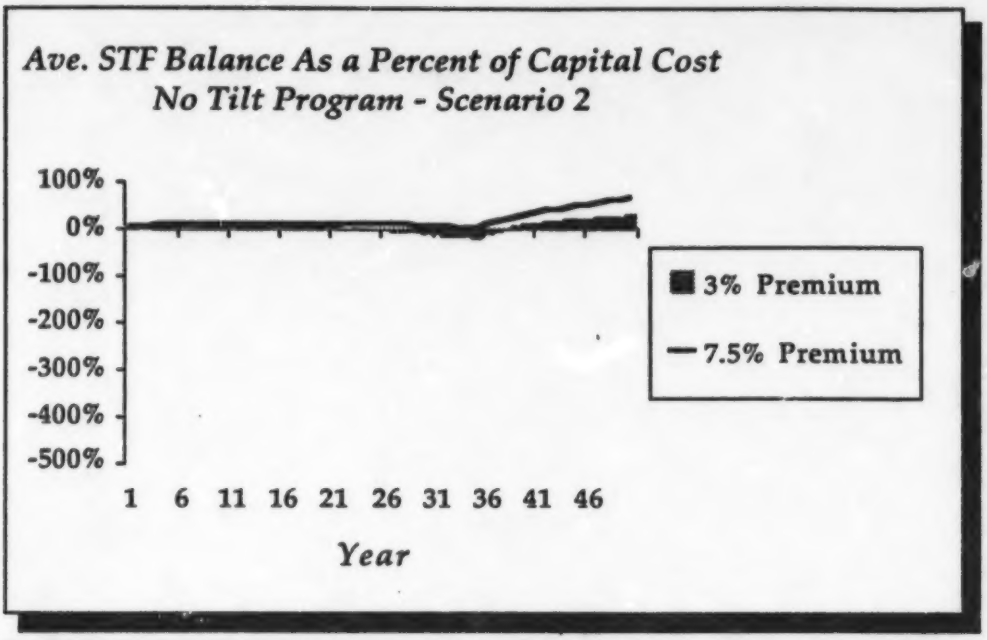


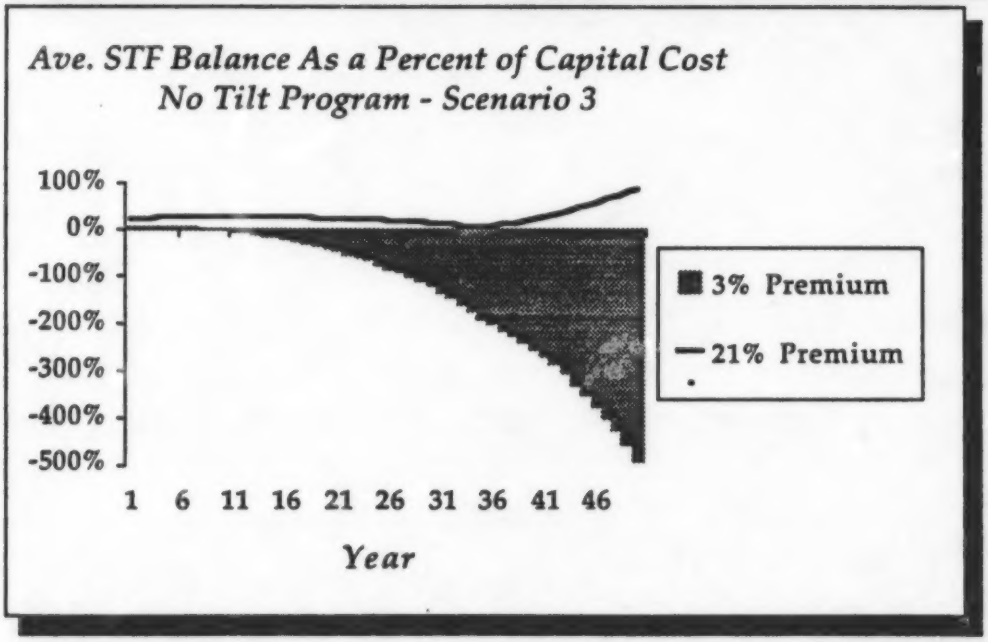

Ave. MIF Cashflow As a Percent of Capital Cost
No Tilt Program
0.00%
-5.00%
-10.00%
-15.00%
-20.00%
1
6
11
16
21
26
31
36
41
46
Year
Scenario 3
Scenario 2

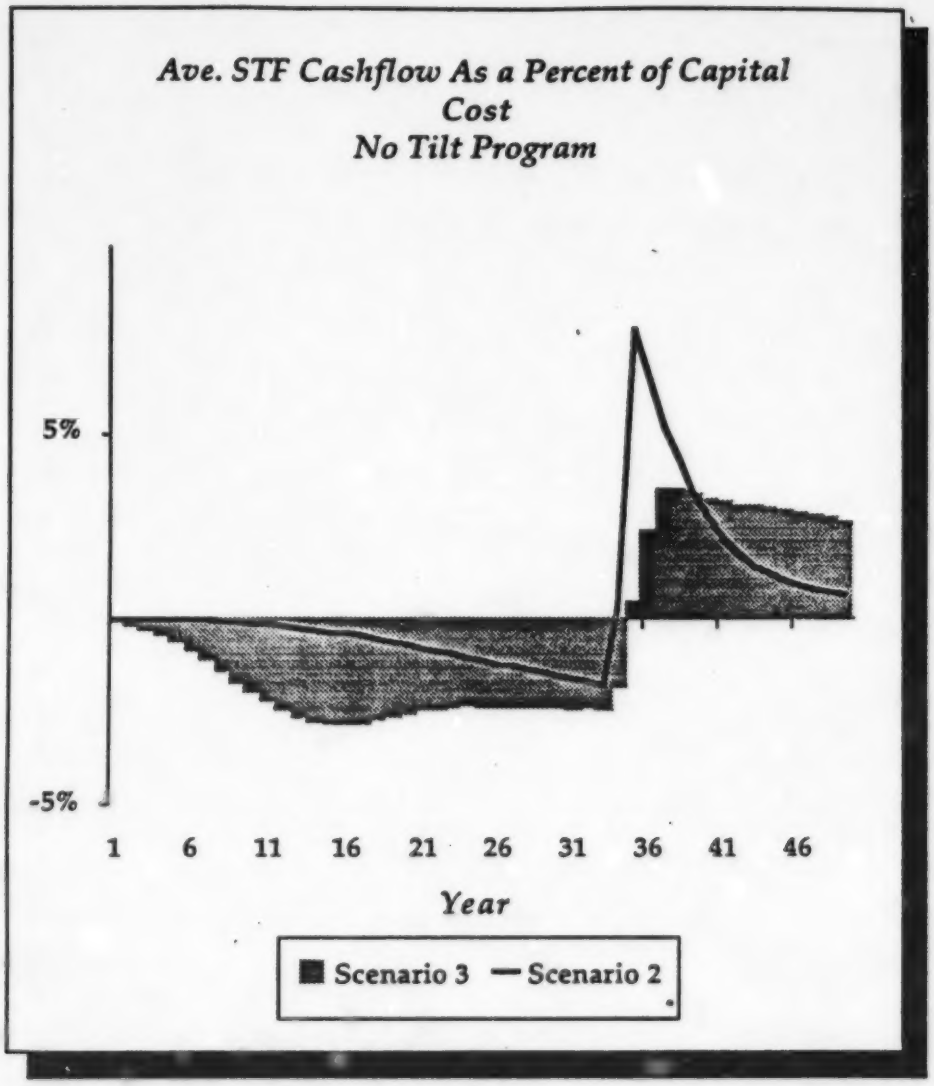

Ave. STF Balance As a Percent of Capital Cost
MRI Program - Scenario 2
100%
75%
50%
25%
0%
1 6 11 16 21 26 31 36 41 46
Year
3% Premium
2.5% Premium


Ave. STF Balance As a Percent of Capital Cost
MRI Program - Scenario 3
200%
0%
-200%
-400%
-600%
-800%
-1000%
1 6 11 16 21 26 31 36 41 46
Year
3% Premium
34.5% Premium


Ave. STF Cashflow As a Percent of Capital Cost
MRI Program
0%
-10%
-20%
-30%
-40%
1
6
11
16
21
26
31
36
41
46
Year
Scenario 3
Scenario 2


Ave. STF Balance As a Percent of Capital Cost
No Tilt Program - Scenario 2
100%
0%
-100%
-200%
-300%
-400%
-500%
1 6 11 16 21 26 31 36 41 46
Year
3% Premium
7.5% Premium


Ave. STF Balance As a Percent of Capital Cost
No Tilt Program - Scenario 3
100%
0%
-100%
-200%
-300%
-400%
-500%
1 6 11 16 21 26 31 36 41 46
Year
3% Premium
21% Premium


Ave. STF Cashflow As a Percent of Capital Cost
No Tilt Program
5%
-5%
1
6
11
16
21
26
31
36
41
46
Year
Scenario 3
Scenario 2